REPORT ON "A THEORY OF STATIONARY TREES AND THE BALANCED BAUMGARTNER-HAJNAL-TODORCEVIC THEOREM FOR TREES"

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The proposed thesis [1] presents work in the mathematical discipline known as Combinatorial Set Theory. The two fundamental concepts forming the focus of the thesis are the notions of a *stationary tree* (treated in Chapter 3), and of a balanced partition relation (treated in Chapter 4). The heart of the thesis is the generalization of a partition relation due to Baumgartner-Hajnal-Todorčević from its context of well-ordered sets to the context of partially ordered sets.

The candidate proves a beautiful theorem (Theorem 29) using a real tour de force that required generalizing the classical theory of stationary sets and regressive functions for ordinals to the context of trees. The exposition of the results presented in the thesis is very thorough, and largely self contained. The candidate also demonstrates exemplary scholarship through his review of the state of the topic preceding the advances made in the thesis. The candidate also demonstrates a thorough and mature knowledge of the literature and the true sources of prior results in this field.

The techniques used in the thesis in advancing the state of knowledge are impressive. The author develops a theory of stationary trees which undoubtedly will infiltrate set theory and its applications fruitfully, as demonstrated by the proof of the fundamental theorem (Theorem 29) of the thesis. The author's thorough treatment of elementary submodels in sections 4.6 and 4.7 is a valuable contribution to the knowledge bank of the field.

There are surprisingly few typographical errors in the thesis. I found only two, both innocuous, and listed after this assessment.

I think that it may be useful to give a summary formulation the main result of the thesis (Theorem 29) and the accompanying investigation of optimality of its hypotheses in a single statement similar to the following:

Theorem 1. *Let κ be an infinite regular cardinal number. Let $(\mathsf{T},<)$ be a tree. Then the following are equivalent:*

1. $(\mathsf{T},<) \longrightarrow (2)^1_{2^{<\kappa}}$
2. $(\mathsf{T},<) \longrightarrow (2^{<\kappa})^1_{2^{<\kappa}}$
3. $(\mathsf{T},<) \longrightarrow (\kappa+1,\, \kappa)^2$
4. *For each positive integer k and for each ordinal ξ such that $2^{|\xi|} < \kappa$,*

$$(\mathsf{T},<) \longrightarrow (\kappa+\xi)^2_k$$

Here (1) is the statement that the tree is not $2^{<\kappa}$-special, and is equivalent to (2). That (3) implies (2) is Corollary 56. (4) evidently implies (3). The main work, the central result of the thesis, is that (2) implies (4).

Conclusion

This thesis meets, and in several aspects exceeds, all expectations of what constitutes a Doctoral Dissertation in Mathematics. The author has demonstrated in writing a level of professionalism and accomplishment befitting of the academic title "Doctor". I recommend that this thesis be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Typographical errors

Page 1 Paragraph 2, sentence 2: *However, The* should be *However, the*.

Page 52 In the proof of Lemma 96, in the sentence beginning *More precisely:*, the word *and* is misspelled as *ans*.

References

[1] A.M. Brodsky, *A theory of stationary trees and the balanced Baumgartner-Hajnal-Taylor theorem for trees*, manuscript of February, 2014.