Abstracts for the Conference on Representations of Algebras, Groups and Semigroups

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Elementary abelian subgroup induction and some applications to cohomology of infinite groups

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Elementary abelian subgroup induction plays a crucial role in cohomology and representation theory of finite groups. Roughly speaking, the results say that important cohomological properties hold for a group ring $R\Gamma$, Γ finite and R an arbitrary ring (with 1), if and only if they hold for RE where E runs over all elementary abelian subgroups of Γ (for instance a theorem of Chouinard says that a module M over a modular group algebra $k\Gamma$ is projective if and only if M is projective over kE where E runs over all elementary abelian subgroups of Γ). In general, similar statements are false if one replaces the family of elementary abelian subgroups by cyclics. Using crossed products and profinite completions we apply this theory to cohomology of infinite groups.

Title: Congruence Borel Orbits of Symmetric Matrices - A Combinatorial Approach

Speaker: Eli Bagno, Bar-Ilan University

Abstract: We deal with orbits of a congruence action of Borel matrices on the algebra of symmetric matrices. We present a natural order on these orbits and compute the rank function of the corresponding poset, using permutation statistics such as the excedence number and the number of inversions.

Hidden Symmetries of Invariant Distributions and representation theory.

Joseph Bernstein

Abstract.

In this talk I will give a historical account of a series of results about invariant distributions and their applications to representation theory.

First I discuss the case of finite groups and show the applications of Gelfand's trick (introduced in 1950) to representation theory.

Then I discuss modifications of Gelfand's method due to Gelfand and Kazhdan which allow to establish similar results for groups over real and p-adic fields.

I will finish with a description of the recent proof of the multiplicity one conjecture by A.Aizenbud, D.Gourevitch, S.Rallis and G.Schiffmann.

Title: Table Algebras: Aliases and Applications

Speaker: Harvey Blau, Northern Illinois University

Abstract: The concept of a table algebra has been discovered and rediscovered asurprising number of times over nearly a century. This has resulted indiverse terminology and appearances in a broad range of areas. We discuss the concept and survey some of its applications, both historical and current. These include representations and characters of finite groups and association schemes, conjugacy classes of finite groups, and conformal field theory.

Title: Permutation representations on invertible matrices.

Speaker: Yona Cherniavsky, Bar-Ilan University

Abstract: We discuss permutation representations which are obtained by the natural action of Sym(n) x Sym(n) on itself as its diagonal subgroup on some special sets of invertible (0,1)-matrices, defined by simple combinatorial attributes. We decompose these representations into irreducibles. The multiplicities involved have a nice combinatorial interpretation. We also generalize known results on asymptotic behavior of the conjugacy representation of Sym(n). This is joint work with Eli Bagno.

Title: Some problems involving the symmetric group on three letters.

Speaker: David Chillag, The Technion

Abstract: The symmetric group on three letters is the only finite group in which any two elements of the same order are conjugate. This was proved by Feit- Seitz and Fitzpattrick. It is conjectured that the symmetric group on three letters is also the only finite group in which any two elements with the same conjugacy-class size are conjugate. We will survey variations of these two statements.

Title: Representations , Fourier transforms and a Verlinde-type formula for finite dimensional Hopf algebra.

Speaker: Professor Miriam Cohen, Department of Mathematics, Ben-Gurion University of the Negev

Abstract: We shall describe some recent developments in the theory of Hopf algebras.Some aspects of semisimple Hopf algebras (which are necessarily finite-dimensional) will be dealt with on one hand and we shall describe an appropriate Fourier transform and its application to an algebraic proof of the Verlinde formula. Non-semisimple finite dimensional Hopf algebras which are also symmetric as algebras will be shown to lead to generalizations of these ideas.

Title: Semiclassical limits of quantized coordinate rings

Speaker: Ken Goodearl, University of California-Santa Barbara

Abstract: By now, the ``Cheshire cat'' description of quantum groups is well known -- a quantum group is not a group at all, but something that remains when a group has faded away, leaving an algebra of functions behind. Various attributes of an algebra $A$ that should qualify it as ``a quantum group corresponding to a group G'' have been accumulated, an important one being that an appropriate algebra R of functions on G should support a Poisson structure which records, to first-order in a suitable sense, the noncommutativity of the multiplication in A. One then says that A is a quantization of R. The Poisson algebra R is typically induced from a family of quantizations, and is called the semiclassical limit of the family. These concepts are also important, more generally, for algebras of functions on manifolds, algebraic varieties, and other systems.

The aim of this talk is to introduce the above ideas, present a few examples, and discuss relationships among these concepts.

Title: On a Commuting Graph on Conjugacy Classes in Groups

Speaker: Marcel Herzog, Tel Aviv University

Abstract: If G is a group, let A(G) be the graph with vertices consisting of the non-trivial conjugacy classes of G and two non-trivial classes C and D are connected if there exist c in C and d in D such that cd=dc. We show that if G is either a finite solvable group or an infinite solvable periodic group, then A(G) has at most two components, each of diameter at most 15.
We also investigate finite and periodic non-abelian groups G satisfying one of the following properties: (i) there are no edges between non-central conjugacy classes of G; (ii) there are no edges between finite conjugacy classes of G. Joint work with Patrizia Longobardi and Mercede Maj.

Title: Generic Representation Theory of Quivers with Relations

Speaker: Birge Huisgen-Zimmermann, University of California-Santa Barbara

Abstract: The irreducible components of varieties parametrizing classes of finite dimensional representations of a finite dimensional algebra A are explored, in terms of both their geometry and the modules they encode. In particular, we establish the existence (and uniqueness, in a sense to be specified) of modules which display *all* categorical generic properties of the modules in a given irreducible component V of one of these varieties, and follow with an investigation of their properties; here "categorical" means "stable under self-equivalences of the category of A-modules", and "generic" means "satisfied by all modules corresponding to the points in a dense open subset of V". The sharpest specific results on the various fronts are obtained for truncated path algebras, that is, path algebras of quivers modulo ideals generated by all paths of a fixed length. In this situation, the generic modules and their syzygies can be constructed explicitly from the underlying quiver and the Loewy length of the algebra.

Title: On Duality Inducing Automorphisms and Sources of Simple Modules of Finite Groups.

Speaker: Radha Kessar, University of Aberdeen

Abstract: Let k be an algebraically closed field of prime characteristic p and G a finite group. To every indecomposable kG module, M, is associated a G-conjugacy class of pairs (Q, V), where Q is a p-subgroup of G and V is an indecomposable kQ-module; such a pair is called a vertex-source pair of M. If M is simple, then it is known that in many cases, the source V is an endo-permutation kQ-module, for instance if G is a p-solvable group or if M lies in a nilpotent p-block of kG. My talk will center around the following problem: Given a p-group Q, which indecomposable endo-permutation kQ modules V have the property that (Q,V) is a vertex source pair of a simple module of a finite group?
I will present some results that indicate that the presence of duality inducing automorphisms in finite classical groups force such modules V to have finite order in the Dade group of Q.

Title: Hecke operators for groups over 2-dimensional fields.

Speaker: David Kazhdan, Hebrew University

Abstract: If G is a reductive group over a one-dimensional field $F = \mathbb{F}_q((t))$ one can define the convolution $\star$ on the space $\mathcal{H}$ of two-sided $G(\mathcal{O})$-invariant $\mathbb{C}$-valued functions on $G(F)$ with compact support. Moreover the spherical Hecke algebra $(\mathcal{H}, \star)$ is isomorphic to the Grothendick ring of finite-dimensional representations of the dual group ${}^L G$. I'll discuss a definition and a description of the Hecke algebra for two-dimensioal fields.

Application of the Representation Theory of Symmetric Groups for the Computation of Chromatic Polynomials of Graphs.

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Chromatic polynomials form one of the most important concepts in graph theory, because they encode many important properties (the number of proper vertex colourings with x colours, the number of acyclic orientations, etc.; for more information on chromatic polynomials see the recent book [1]).

Computing the chromatic polynomial of graphs is an important algorithmic problem. However, already the problem of deciding whether a given graph has a proper 3-colouring is NP-complete. Hence in general it can not be expected to give ONE algorithm that is going to be efficient for ALL graphs. Instead algorithms are searched for, that are efficient for interesting classes of graphs (polynomial algorithms are known e.g. for complete graphs, trees, cycles, ladders, outer-planar graphs, graphs of bounded tree-width, graphs of bounded clique-width,...).

In this talk we will outline the relatively recent idea to apply the representation theory of symmetric groups in order to compute the chromatic polynomials of so called generalized ladder graphs. The described method is effective and its running time is linear in the number of rungs of the generalized ladder graph.

This talk is based on ideas originally developed by the authors during a visit of CP to Israel in the year 2000 ([2]). Meanwhile the ideas inspired a series of papers dealing with so called bracelet-graphs ([3, 4]).

References

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[2] M. Klin, Ch. Pech. *Chromatic polynomials and coherent configurations: an interplay between two well-known approaches in algebraic graph theory.* Unpublished manuscript, 2000.

[3] N.L. Biggs, M.H. Klin and P. Reinfeld. *Algebraic methods for chromatic polynomials.* Eur. J. Comb. **25**(2), 2004, 147–160.

[4] N.L. Biggs. *Specht modules and chromatic polynomials.* J. Combinatorial Theory (B) **92**, 2004, 359–377.

[1]The author was supported by the Skirball postodctoral fellowship of the Center of Advanced Studies in Mathematics at the Mathematics Department of Ben Gurion University.

Title: Elements of Spectral Theory in Algebras

Speaker: Yakov Krasnov, Bar-Ilan University

Abstract: We establish the basic properties for the set of idempotents in finite dimensional non-associative real m-ary algebras using the syzygies among the Peirce numbers as spectral parameters. We study the reciprocal influence of the Peirce numbers allocation on the geometry of the subalgebras. The canonical form of m-ary algebras multiplication and theirs tensors representations will be present.

Title: Vertices of simple modules for symmetric groups

Speaker: Burkhard Kuelshammer, Jena

Abstract: Let F be an algebraically closed field of characteristic p>0, and let G be a finite group. Then every indecomposable FG-module M determines a conjugacy class of p-subgroups of G called the vertices of M. These vertices measure the relative projectivity of M; in particular, M is projective if and only if the trivial subgroup of G is a vertex of M. On the other hand, the vertices of the trivial FG-module F are the Sylow p-subgroups of G. In my talk, which is a report on the work of my former students S. Danz, B. Fotsing and R. Zimmermann, I will indicate what is known about vertices in the case where G is a finite symmetric group and M is simple, and I will state a number of open questions.

Title: Generalized reflections and derived equivalences of posets

Speaker: Sefi Ladkani, Hebrew University

Abstract: A finite partially ordered set (poset) X carries a natural structure of a topological space. This allows us to identify, for any abelian category A, sheaves over X (with values in A) and commutative diagrams (over A) whose shape is the Hasse diagram of X. When A is the category of vector spaces over a field, these can also be identified with right modules over the incidence algebra of X, thus providing a link with the representation theory of algebras.

We say that two posets are derived equivalent if their bounded derived categories of sheaves are equivalent as triangulated categories. This leads to an equivalence relation between posets, which is strictly coarser than isomorphism, but still fine enough to be interesting. However, there is no known algorithm that determines for two posets whether they are derived equivalent or not.

I will start by explaining the above notions and briefly discussing combinatorial invariants of derived equivalence, that is, properties of a poset that are shared by all other posets derived equivalent to it. Then I will present several constructions that systematically produce, given a poset, new posets derived equivalent to it. The common theme of these constructions is the structured reversal of order relations.

One of these constructions consists of new generalized reflection operations, which are used to prove that a poset and its reflection are derived equivalent. I will describe these generalized reflections in explicit combinatorial terms and show how they generalize the usual Bernstein-Gelfand-Ponomarev reflection functors originally introduced for the representations of quivers.

Title: Representation Growth: The Local Case

Speaker: Michael Larsen, Indiana University

Title: On Graded Centres of Triangulated Categories and Block Cohomology

Speaker: Markus Linckelmann

Abstract: Any triangulated category gives rise to a graded ring which generalises the notion of the centre of a category. When applied to derived categories of blocks of group algebras this notion is closely related to block cohomology. More mysterious are graded centres of stable categories of block algebras which one would expect to be closely related to the Tate analogue of block cohomology - but it turns out, using almost split sequences, that the degree -1 component of the stable category of a block with infinite representation type is infinite dimensional.

Title: Representation growth of arithmetic groups

Speaker: Alex Lubotzky, Hebrew University

Abstract: Let D be a finitely generated group and Rn(D) the number of its n-dimensional complex irreducible representations. Rn(D) may be infinite but it is finite for higher rank arithmetic groups. In fact B. Martin and the speaker showed that the sequence grows polynomially if D satifies the congruence subgroup property. We will present more recent result (joint work with M. Larsen) which gives futher information on the sequence and the "zeta function" associted with it.

Title: On Finite Bases for Principal Factors in Full Transformation Semigroups

Speaker: Grigori Mashevitsky, Ben-Gurion University of the Negev

Abstract: The semigroup $T_k(X)$ of transformations of rank at most k of a set X has no finite basis of identities if and only if either $k = 2$ and $|X| \in \{3,4\}$ or $k \geq 3$.

Faithful linear representations of bands

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A semigroup S such that $a^2 = a$ for every $a \in S$ is called a band. The main motivating problem for this talk is to find conditions on a band S in order that S embeds into the multiplicative semigroup $M_n(F)$ of $n \times n$ matrices over a field F for some $n \geq 1$. It is known and easy to show that this is always the case if S is a rectangular band (that is, a semigroup satisfying the identity $xyx = x$), but this is no longer true in general (that is, in case the band S has at least 2 rectangular band components). The following related problem will be also discussed: when the semigroup algebra $K[S]$ of a band S over a field K is embeddable into $M_n(A)$ for a commutative algebra A? Certain general results will be proved and some concrete embeddings will be constructed. This talk is based on a joint work with Ferran Cedo.

Connections between semigroup and group representations

Mohan S. Putcha

ABSTRACT

Historically semigroup representation theory has developed independently of group representation theory. We will present some viewpoints that yield some strong connections between the two theories. We will discuss how semigroup representation theory can yield some new concepts, directions and techniques for group representation theory, in particular new forms of induction and new concepts of weights.

Varieties Generated by Completely 0-Simple Semigroups

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Abstract

A *completely 0-simple semigroup* is just a semigroup with a zero, no ideals other than S and $\{0\}$ and containing an idempotent element e ($e^2 = e$) that is primitive ($f^2 = f, fe = ef = f \Longrightarrow f = e$). The class of completely 0-simple semigroups was one of the first classes of semigroups to be analysed in depth (by Rees and Sushkevich) in the 1930s and, as one of the most important building blocks for semigroups, completely 0-simple semigroups have played a fundamental role in the theory of semigroups ever since. The varieties generated by individual completely 0-simple semigroups have been the focus of considerable interest over the years, but it was only with the determination of a basis of identities for the variety $\mathbf{RS_n}$ generated by all completely 0-simple semigroups with subgroups of exponent dividing n (by T.E.Hall, S.I.Kublanovsky, S.Margolis, M.Sapir and P.G.Trotter in 1997) that a general study of the varieties generated by completely 0-simple semigroups could be undertaken. This was initiated by S.I. Kublanovsky. In this talk I will discuss recent results regarding the the lattice of subvarieties of $\mathbf{RS_n}$ and the characterization of those subvarieties of $\mathbf{RS_n}$ that are generated by completely 0-simple semigroups.

Title: Prime Decomposition Theorem for Finite Idempotent Semirings Using the Triangular Product of B. I. Plotkin

Speaker: John Rhodes, University of California-Berkeley

Abstract: A Prime Decomposition Theorem for finite idempotent semirings is proved using the triangular product of Plotkin adapted to semirings. A pair of results referred to as the Triangular Decomposition Theorem and the Ideal Decomposition Theorem are presented. Applying these in the context of idempotent semirings yields the decomposition half of the Prime Decomposition Theorem for idempotent semirings.

Further portions of the talk are devoted to proving matrix algebras over the power set of a finite group are irreducible with respect to the triangular product. A moral of the talk is that much more of ring theory works over semirings than one might expect. Applications to computing group complexity of the power set of a finite semigroup are given.

This is new joint research with Benjamin Steinberg and is covered in Chapter 9 of our book 'The q -Ttheory of Finite Semigroups' Springer 2008.

UNIFORMLY CONTINUOUS FUNCTIONS FOR SOME PROFINITE TOPOLOGIES

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Given a pseudovariety $\mathbf{V}$ of finite monoids, the pro-$\mathbf{V}$ quasi-metric $d_{\mathbf{V}}$ can be defined on an arbitrary monoid M through

$$d_{\mathbf{V}}(u,v) = 2^{-r_{\mathbf{V}}(u,v)},$$

$$r_{\mathbf{V}}(u,v) = \min\{|N| \mid N \text{ is in } \mathbf{V} \text{ and separates } u \text{ and } v\}$$

with the standard conventions. Clearly, $d_{\mathbf{V}}$ is a metric if and only if M is residually in $\mathbf{V}$. We say that a mapping of monoids $f : M \to N$ is

- $\mathbf{V}$-*uniformly continuous* if f is uniformly continuous for the pro-$\mathbf{V}$ quasi-metric on both M and N;
- $\mathbf{V}$-*hereditarily continuous* if f is $\mathbf{W}$-uniformly continuous for any subpseudovariety $\mathbf{W}$ of $\mathbf{V}$.

The importance of uniform continuity in the context of language theory relies on the fact that the mapping $f : M \to N$ is $\mathbf{V}$-uniformly continuous if and only if f^{-1} preserves $\mathbf{V}$-recognizable subsets.

Let $\mathbf{G}$ denote the pseudovariety of all finite groups and let $\mathbf{G}_p$ denote the pseudovariety of all finite p-groups for a given prime p. Borrowing binomial decompositions from p-adic analysis in the case of integers and producing their generalizations for words, we shall present several results concerning $\mathbf{G}_p$ and $\mathbf{G}$-uniformly (hereditarily) continuous mappings of the form $f : M \to \mathbb{Z}$, where M is a free monoid, a free commutative monoid or a free commutative group of finite rank.

Title: Mobius functions and semigroup representation theory

Speaker: Benjamin Steinberg, Carleton University

Abstract: Using Rota's theory of Mobius inversion, we are able to make very explicit the work of Munn and Ponizovskii on representations of inverse semigroups. In particular, one can obtain a formula for multiplicities of representations using only knowledge of the characters of maximal subgroups and the Mobius function of the idempotent semilattice. Since most important inverse monoids, such as Renner monoids of algebraic monoids, have Eulerian semilattices, this leads to relatively simple formulas.

The results for inverse semigroups can be made to work for other classes of semigroups including semigroups of upper triangular matrices over a field. This leads to applications in computing spectra of random walks on such semigroups.

Visual representation of digraph

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The main topic of the investigation is the visual representation of the transition graph of an automaton in the graphical form on the base of structure properties of the graph. The matrix representation could not demonstrate the structure of the automaton, it can be done better by the help of the graphical facilities.

One of the most popular programs in the area of the visualization of a graph is the package GRaphViz (http://home.so-net.net.tw/oodtsen/wingraphviz), a project of AT&T Labs research. It provides a collection of tools for generating graph layouts.

The package TESTAS (http://www.cs.biu.ac.il/∼trakht/syn.html) tested the properties of GRaphViz. Unfortunately, the time of the creation of the image was too long. It seems that the complexity of the algorithm is even not polynomial. The edges are curved, the properties of the automaton are not demonstrated, therefore clearness of the graph is far from desired. The approach of GRaphViz seems too general and does not use the specificity of the transition graph of an automaton.

Therefore for to remove the shortages of visualization packages similar to GRaphViz we consider another approach paying attention mostly to the structure properties of the automaton and to the complexity of used algorithms. The consideration is restricted to the class of the transition graphs of the deterministic finite automata, a very private case of a graph, so some possible restrictions can be used. The graph is presented as a union of the set of the strongly connected components (SCC). A linear algorithm for finding SCC is implemented. The vertices of every SCC form a cycle in the graph layout. All SCC are ordered according to the *weight* defined by the size of SCC and the number of ingoing edges. The edges of the graph with different labels differ by its colors, so the labels of the edges can be omitted.

The pictorial diagram demonstrates the inner structure of the graph and is clear illustration of the graph presented by its SCC. The traditional problem of placing of the states with their labels and the edges together with labels is solved on the base of a linear algorithm. However, the placing of the objects is not the most important problem on the list of our tasks.

The algorithm is implemented in C++ as a part of the package TESTAS. The linearity of the implemented algorithms ensures the momentary appearance of the representation.

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