Application of the Representation Theory of Symmetric Groups for the Computation of Chromatic Polynomials of Graphs.

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Chromatic polynomials form one of the most important concepts in graph theory, because they encode many important properties (the number of proper vertex colourings with x colours, the number of acyclic orientations, etc.; for more information on chromatic polynomials see the recent book [1]).

Computing the chromatic polynomial of graphs is an important algorithmic problem. However, already the problem of deciding whether a given graph has a proper 3-colouring is NP-complete. Hence in general it can not be expected to give ONE algorithm that is going to be efficient for ALL graphs. Instead algorithms are searched for, that are efficient for interesting classes of graphs (polynomial algorithms are known e.g. for complete graphs, trees, cycles, ladders, outer-planar graphs, graphs of bounded tree-width, graphs of bounded clique-width,...).

In this talk we will outline the relatively recent idea to apply the representation theory of symmetric groups in order to compute the chromatic polynomials of so called generalized ladder graphs. The described method is effective and its running time is linear in the number of rungs of the generalized ladder graph.

This talk is based on ideas originally developed by the authors during a visit of CP to Israel in the year 2000 ([2]). Meanwhile the ideas inspired a series of papers dealing with so called bracelet-graphs ([3, 4]).

References

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[1]The author was supported by the Skirball postodctoral fellowship of the Center of Advanced Studies in Mathematics at the Mathematics Department of Ben Gurion University.