UNIFORMLY CONTINUOUS FUNCTIONS FOR SOME PROFINITE TOPOLOGIES

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Given a pseudovariety $\mathbf{V}$ of finite monoids, the pro-$\mathbf{V}$ quasi-metric $d_{\mathbf{V}}$ can be defined on an arbitrary monoid M through

$$d_{\mathbf{V}}(u,v) = 2^{-r_{\mathbf{V}}(u,v)},$$

$$r_{\mathbf{V}}(u,v) = \min\{|N| \mid N \text{ is in } \mathbf{V} \text{ and separates } u \text{ and } v\}$$

with the standard conventions. Clearly, $d_{\mathbf{V}}$ is a metric if and only if M is residually in $\mathbf{V}$. We say that a mapping of monoids $f : M \to N$ is

- $\mathbf{V}$-*uniformly continuous* if f is uniformly continuous for the pro-$\mathbf{V}$ quasi-metric on both M and N;
- $\mathbf{V}$-*hereditarily continuous* if f is $\mathbf{W}$-uniformly continuous for any sub-pseudovariety $\mathbf{W}$ of $\mathbf{V}$.

The importance of uniform continuity in the context of language theory relies on the fact that the mapping $f : M \to N$ is $\mathbf{V}$-uniformly continuous if and only if f^{-1} preserves $\mathbf{V}$-recognizable subsets.

Let $\mathbf{G}$ denote the pseudovariety of all finite groups and let $\mathbf{G}_p$ denote the pseudovariety of all finite p-groups for a given prime p. Borrowing binomial decompositions from p-adic analysis in the case of integers and producing their generalizations for words, we shall present several results concerning $\mathbf{G}_p$ and $\mathbf{G}$-uniformly (hereditarily) continuous mappings of the form $f : M \to \mathbb{Z}$, where M is a free monoid, a free commutative monoid or a free commutative group of finite rank.