and if feature through $V_i$. If $\lambda \neq 0$, then we get a map

$$\phi _{i} \to W$$

For an generic $l$, the source of this map is an irreducible principal series, hence $\phi _{i}$ is injective.

**Def** let $X_1(\sigma _i)$ be the "minimal" quotient of $\sigma _i$.

Let

$$\tau _i = \phi _i(X_1(\sigma _i)) \in V_i$$

Then $\tau _i$ quotient an irreducible $K^2$-module in $V_i$, isomorphic to $\sigma _i$.

**Def** A map $V_0 \to V_{i-1} \to W$ is annulled if $\tau (\tau _i) = 0$ for all $i$.

Then if $W$ has good rank and $\tau : V_i \to W$ is annulled and surjective, then $W$ is irreducible.

**Proof** let $U \subseteq W$ be an irreducible $G$-module. Since $X_0$ quotient $V_0$, it suffices to show $\tau (X_0) \notin U$.

Consider rank of $U$, some $\tau (X_1) \notin U$, hence $\tau (\tau _i) \notin U$ as $X_1$ quotient image of $\phi _i$.

By multiplication one in rank if $W$, $\tau (Z_1) = c_i \tau (X_1)$ for some scalar $c_i \neq 0$, hence $\tau (X_1) \notin U$. Hence...

End The family of B-P representations is parametral...
by parameter in \( F_p \). We expect that amenability
comparative to simultaneous non-vanishing of
a finite set of polynomials in these parameters,
then build-Pathmanup an quasi-nilpotent

and Amenable condition is sufficient but
not necessary. Work of A. Mera on real
filtration of \( \text{ind}^{\text{ker} \Gamma} \) get fractal behavior.

Question: let \( c_0, \ldots, c_{e-1} \in \overline{F}_p \) and consider

\[
B(\varepsilon) = \sqrt{\langle z_1 - c_1 x_0, \ldots, z_{e-1} - c_{e-1} x_{e-2} \rangle}
\]

Is this admissible?

Any quotient with good scale in inseparable?
What are they?
Lecture 2

Weights in class conjecture for totally real fields

1. Conj (Shen, 1970's). Let \( \phi: \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{Cl}_e(\overline{F}_p) \) be a continuous, sinistrally, and cdlt \( \phi|_{\text{cdlt}} = -1 \)

Galois representation. Then \( \phi = \phi_\mu \) for \( \phi_\mu \) modular from and \( \phi_\mu \) the Galois up arising from it by the Eichler-Shimura-DeRigne construction.

Theorem also specified the weights of \( \phi_\mu \).

The full conjecture is a then of Khare-Wintenberger-Kisin. The implication of modular \( \Rightarrow \)

\( \phi_\mu \) modular of weight \( \mu \) were known earlier. Deligne, Fontaine, etc. Will see examples of such theorem later.

Let \( p \) be prime, \( F \) totally real field,

\[ p \mathcal{O}_F = \mathfrak{p}_1^{e_1} \cdots \mathfrak{p}_r^{e_r}. \]

Def. A finite weight \( \tilde{\mathfrak{w}} \) in an sinistrally \( \overline{F}_p \)-up of \( \text{Cl}_e(\mathcal{O}_F/p) \). Thence factors through \( \prod_{i=1}^{r} \text{Cl}_e(\mathcal{O}_F/\mathfrak{p}_i) \)

A finite weight \( \tilde{\mathfrak{w}} \) up \( \overline{F}_p \)-up of \( \text{Cl}_e(\mathcal{O}_F/p) \).

We will define what it means for \( \phi \) to be modular of a finite weight \( \tilde{\mathfrak{w}} \). For each \( \mathfrak{p} \mid p \)

will define a set \( W_{\phi}(p) \) of finite weights \( \tilde{\omega} \),

determined by \( \phi|_{\tilde{\mathfrak{w}}|_{\mathfrak{p}}} \), such that...
The nodal weights of \( \mathfrak{p} \)

Let \( \mathfrak{p} : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \text{Gal}(\overline{\mathbb{F}_p}) \) be continuous, and totally odd, then it must be nodal of weight

\[
W(\mathfrak{p}) = \{ \sigma = \bigotimes g_p : g_p \in W_p(\mathfrak{p}) \}.
\]

If \( \mathfrak{p} \) is not tame, expect the same structure but with \( W_p(\mathfrak{p}) \neq W_p(\mathfrak{p}^{\text{ss}}) \).

**Conjecture**

\( F = \mathbb{Q} \) line

- \( F \) totally real, \( p \) minus BOS
- \( F \) totally real, \( p \) tame MMS

\( \text{Gal}_n, \ n \geq 3, \ F = \mathbb{Q}, \ p \) tame And et al.

All, but few explicit Greg.

Fix \( q \mid p \), let \( h_q = F_q = \mathbb{F}_q \). The line weights at \( q, q \)-adic:

\[
\bigotimes_{\tau : h_q \to \mathbb{F}_p} \text{det} W_2(\text{sym}^2 h_q \otimes \tau^* F_q)\]

\( 0 \leq \tau_2 \leq p-1 \)

\( 0 \leq w_2 \leq p-1 \).

Let \( \tau : F_q^\times \to \mathbb{F}_p \). Let \( F_q^{\text{wild}} \) be the wild inert. Let \( \omega_q \) be the fundamental character of \( \text{min} \) \( n_i \).

\[
\omega_{n_2} : F_q \to F_q / \mathfrak{p}_q \xleftarrow{\text{lin}} F_q^{\text{wild}} \xrightarrow{\tau} \mathbb{F}_p.
\]
If \( \mathfrak{p} \) is totally ramified, then \( \mathfrak{p} | \mathfrak{p}_q \) factors through \( \mathbb{F}_q / \mathbb{F}_p \). 

Let \( \mathfrak{p} \) act by conjugation. Then in addition, \( \mathfrak{p} | \mathfrak{p}_q = \mathfrak{q} \otimes \mathfrak{q}' \). 

Case 2: \( \mathfrak{p} \) is reducible, \( \mathfrak{q} = \mathfrak{q}' \) of union \( \mathfrak{p} \).

Case 1: \( \mathfrak{p} \) is irreducible, \( \mathfrak{q} = \mathfrak{q}' \), \( \mathfrak{q}' \) of union \( \mathfrak{p} \).

Let \( e \) be the ramification index of \( \mathbb{F}_q / \mathbb{F}_p \). Then:

1) In reducible case:

\[
\sigma = \otimes_{\mathfrak{p} \to \mathfrak{q}} \omega_{\mathfrak{p}} \otimes_{\mathfrak{q} \to \mathfrak{q}'} \omega_{\mathfrak{q}'}
\]

\( \sigma \in W_\sigma(p) \) if and only if for each \( \mathfrak{q} \in \mathcal{I} = \mathcal{I}_\mathfrak{p} : \mathfrak{q} \to \mathfrak{q}' \) there exists \( \mathcal{T} : \mathbb{F}_q^2 \to \mathbb{F}_p \) left-

shift \( \tau \) and an integer \( 0 \leq s_2 \leq e-1 \) such that

\[
\begin{pmatrix}
\mathbb{F}_q^2 & 0 \\
0 & \mathbb{F}_p
\end{pmatrix}
\]

2) If \( \mathfrak{p} | \mathfrak{p}_q \) is reducible, then \( \sigma \in W_\sigma(p) \) if and only if in some subset \( S \in \mathcal{I} \) and \( 0 \leq s_2 \leq e-1 \) for all \( \mathfrak{q} \in \mathcal{I} \) such that

\[
\mathfrak{p} | \mathfrak{p}_q \sim \prod_{\mathfrak{q} \in S} \omega_{\mathfrak{q}} \prod_{\mathfrak{q} \not \in S} \omega_{\mathfrak{q}'}
\]

Rule: 1) \( W_\sigma(p) \) should be viewed as a multiset, where the multiplicity of \( \sigma \) is the number of
different collection of $\mathbb{Z}_p$ that gives rise to it.

Note the possible $\pi_{F,p}$ depend only on the
morphism field $h_\theta$, so they are the same for
$Q < F_0 < F$ when $\theta$ remains non-nilpotent. The
weights coming from $\pi_{F,p}$ give $\mathbb{Z}_p$ a
complete set of modular weights for a lift of
$G_{\mathbb{Q}}$.

Yet another way to look at this: for
simplicity assume there is only one prime $p$ of $F$
laying above $p$.

$$T = \{ \sigma : \mathbb{Q}_p \to \mathbb{Q}_p \}$$

$$I = \{ \tau : \mathbb{Q}_p \to \mathbb{Q}_p \}$$

$151 = e|I|.$

Then the cohomology $H$ is modular of weight
$s = \bigotimes \operatorname{det}^* w_c \otimes \operatorname{sign} \mathbb{Q}_p @ \mathbb{Q}_p \otimes \mathbb{Q}_p \otimes \mathbb{Q}_p$ has a crystalline
lift $\tilde{\rho} : \operatorname{Gal}(\mathbb{Q} / \mathbb{Q}_p) \to \mathbb{G}_a(\mathbb{Q}_p)$ with labelled Hodge-
Tate weights $E_{\rho_r, n_0, 0}$, where for each $r \in T$,

$$E_{w_2, n_0, 0} = \begin{cases} E_{w_2, w_2 + r + 1} & \text{for one } r \text{ alone} \\ E_0, 13 & \text{for the other.} \end{cases}$$

As mentioned yesterday, this conjecture (with
multiplicity) specifies the $K_2$-cohomology of $\pi(F)$,
the $\pi_0$ of $\mathbb{G}_a(\mathbb{Q}_p)$ associated to $\pi_{F,p}$ by the
local $p$-adic Hasse-Weil correspondence.

Results towards the conjecture:

They (Fontaine) suppose that $F = \mathbb{Q}$ and $p \nmid p$ is

invariant. If $p \nmid \pi_{F, \theta}$ for a modular form

$f$ of weight $2 \leq r \leq p+1$, then