Infinitesimal analysis 88-503 homework set 6

Due Date: 14 july '24

1. Let $\langle S_n : n \in \mathbb{N}\rangle$ be a sequence of internal subsets of ${}^*\mathbb{R}$ and let $A = \cap_{n\in\mathbb{N}} S_n$. Is A necessarily internal?

2. Consider the LSEQ transformation (see Section 6.9 of the class notes). Apply LSEQ to Ψ in the following formula and determine whether the new formula is true: $(\forall \epsilon \in \mathbb{R}^+)\, \Psi(\epsilon)$, where $\Psi(\epsilon)$ is the formula

$$(\forall x \in (0,1))\, (\exists \delta \in \mathbb{R}^+)(\forall y \in (0,1))\, \big(|x-y| < \delta \ \rightarrow\ \ln x - \ln y < \epsilon\big).$$

3. Apply LSEQ to Φ in the following formula and determine whether the new formula is true: $(\forall \epsilon \in \mathbb{R}^+)\, \Phi(\epsilon)$, where $\Psi(\epsilon)$ is the formula

$$(\forall x \in (1,\infty))\, (\exists \delta \in \mathbb{R}^+)(\forall y \in (1,\infty))\, \big(|x-y| < \delta \ \rightarrow\ \ln x - \ln y < \epsilon\big).$$

4. Let X be an internal subset of ${}^*\mathbb{R}$. Prove that if X has arbitrarily large finite members, then X has a positive unlimited member.

5. Let $c \in {}^*\mathbb{R}$. Use the internal set definition principle to show that the set $\{b \in {}^*\mathbb{R} : c \le b\}$ is internal.