June 2, 2024

INFINITESIMAL ANALYSIS 88-503 HOMEWORK SET 3

Due Date: 16 june '24

1. Let $\langle r_n : n \in \mathbb{N} \rangle$ be a sequence in $\mathbb{R}^{\mathbb{N}}$ such that $[r_n] \in {}^*\mathbb{R}$ is a positive hyperreal. Define a sequence $\langle s_n : n \in \mathbb{N} \rangle$ such that $[s_n]^2 = [r_n]$ in ${}^*\mathbb{R}$.

2. Let $Rel_{\mathcal{R}}$ be the full relational structure over $\mathbb{R}$, and let $Rel_{{}^*\mathcal{R}}$ be the corresponding relational structure over ${}^*\mathbb{R}$, as defined in Section 4.1 of the class notes (page 41). Prove that the set $\mathbb{N}$ does not belong to $Rel_{{}^*\mathcal{R}}$.

3. Let ${}^*[0,1]$ be the hyperreal extension of the unit real interval. Prove that ${}^*[0,1]$ contains a positive infinitesimal.

4. Let $s : \mathbb{N} \to \mathbb{R}^+$ be a sequence such that the extended hypersequence ${}^*s : {}^*\mathbb{N} \to {}^*\mathbb{R}^+$ never takes infinitesimal values. Prove that s is bounded away from zero in $\mathbb{R}$.

5. (Optional) Let L denote the set of finite hyperrationals, and let I denote the set of infinitesimal hyperrationals. Determine the quotient L/I.