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A study of mathematical determination through Bertrand's paradox. (English summary)
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This article presents a case study in "the dynamics of determination and instrumentality" in the context of mathematical thinking. The particular case is Bertrand's paradox, which presents three apparently plausible geometrical methods for determining the probability that a randomly chosen chord from a circle is longer than the side of the inscribed equilateral triangle; each of Bertrand's methods determines a different probability. The author not only proposes a solution to the paradox, but claims in addition that the numerical method underlying his solution reveals both presuppositions underlying Bertrand's different results and presuppositions underlying the conclusions of very recent treatments of the paradox. (See [Y. D. Sergeyev, EMS Surv. Math. Sci. **4** (2017), no. 2, 219–320; MR3725242] for a discussion of the significance of the claim that the method is numerical.) The author summarizes as follows the insights gained from studying the dynamics of determination and instrumentality:

"Certain mathematical problems, as well as mathematised empirical problems, occur within an enquiry as objects of investigation calling for symbolic instruments adequate to their character and, thus, capable of tackling them. It may well be the case that a canonical array of instruments should prove insufficient to carry out a successful intervention upon a problem, in which case the forging of new instruments is required if progress in enquiry is to be made." (p. 394)

This remark is more a truism than an insight. It is common knowledge, for instance, that to solve problems involving rates of change, the conceptual and symbolic advances of Newton and Leibniz were required. So if the author has revealed something worthwhile, it must be that he has made progress in resolving Bertrand's paradox.

The alleged resolution of Bertrand's paradox depends upon modeling the random selection of a chord by means of Sergeyev's grossone system, which includes infinite numbers and infinitesimals. The author argues that once it has been numericalized, Bertrand's method employing randomly chosen pairs of points gives the best answer, because it is the only method in which the chords are "homogeneously distributed around the circle" (p. 391). The crucial role the author assigns to Sergeyev's calculus goes unmentioned in the abstract, so this reviewer was taken aback by the sudden appearance of grossones in the third section of the paper—taken aback because the system is controversial, to say the least. Louis Kaufmann (Chicago) says, "there is nothing new in this work except a notation G that represents a large number", and disputes Sergeyev's claim to have achieved insight into the continuum and Riemann zeta hypotheses (see Zbl 1890.03048; see also Mikhail Katz's MR review [MR3725242]).

Until it attains a foothold in the mainstream mathematical community as a means for modeling mathematical and empirical problems, it seems unwise to use the grossone system as a tool for drawing philosophical conclusions. After Newton and (especially) Leibniz forged new instruments for solving problems that involved continuous change, their instruments came under fire for producing paradoxical consequences, as the philosopher Berkeley was happy to point out. But the success of their instruments in application to empirical problems rendered moot the significance of the philosophical complaints.

(For complaints about Berkeley's complaints, see [D. M. Sherry, Stud. Hist. Philos. Sci. **18** (1987), no. 4, 455–480; MR0918087].) Only after we have mainstream examples of progress based upon grossones can we be confident that Sergeyev's innovation illuminates Bertrand's paradox.

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Current version of review. Go to earlier version. *David Moore Sherry*