March 29, 2011

# 88-826 DIFFERENTIAL GEOMETRY
# HOMEWORK SET 3

1. Let $K$ be a field, let $V$ be a vector space over $K$, and let $\Lambda(V)$ be its exterior algebra. Thus for any 1-form $v \in \Lambda(V)$, we have $v \wedge v = 0$. Prove that if the characteristic (me'afyen) of $K$ is different from two, then $v \wedge w = -w \wedge v$ for all 1-forms $v, w \in \Lambda(V)$.

2. Prove that every decomposable (simple) 2-form $\eta$ on $\mathbb{R}^4$ satisfies $\eta \wedge \eta = 0$.

3. Let $A \in \Lambda^2(\mathbb{R}^4)$ be defined by the formula

$$A = e_1 \wedge e_2 + e_3 \wedge e_4. \tag{0.1}$$

Prove that $A \wedge A \neq 0$, and conclude that $A$ is not decomposable (simple).

4. Thinking of the symplectic form $A$ on $\mathbb{R}^4$ as the imaginary part of a Hermitian inner product, prove that the comass norm of $A$ equals 1.

5. Consider the standard flag (degel) in $\mathbb{C}^4$, and consider the corresponding decomposition of $\mathbb{CP}^3$ into cells (ta'im). Let $e^4$ be the 4-dimensional cell of the decomposition. Prove that its closure in $\mathbb{CP}^3$ is a copy of $\mathbb{CP}^2$.

6. On the unit circle $S^1$, consider the standard 1-form traditionally denoted $d\theta$. Prove that $d\theta$ is not a coboundary, i.e. it is not in the image of the differential $d : C^\infty(S^1) \to \Omega^1(S^1)$. (Hint: use Stokes' theorem.)

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