Differential geometry 88-826 homework set 2

Due Date: 7 april '22

1. Let A and B be copies of $\mathbb{R}^3$ with transition function $\phi(v) = \frac{v}{v \cdot v}$ whenever $v \in \mathbb{R}^3 \setminus \{0\}$. Prove that the resulting manifold M with coordinate patches A and B is metrizable.

2. Let $M = T^3$ be the 3-torus. Prove that the tangent bundle of M is diffeomorphic to the product $T^3 \times \mathbb{R}^3$.

3. Let $0 < b < a$. Consider the Jordan curve

$$C = \{(x, z) \in \mathbb{R}^2 : (x - a)^2 + 4z^2 = b^2\}$$

and the corresponding torus of revolution $T_C \subseteq \mathbb{R}^3$. Express the conformal parameter of T_C in terms of an integral (there is no need to evaluate the integral).