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★Husserl and Cantor. (English summary)

Essays on Husserl's logic and philosophy of mathematics, 169–196, *Synth. Libr.*, 384, Springer, Dordrecht, 2017.

Edmund Husserl and Georg Cantor were colleagues and close friends during the last 14 years of the 19th century. In her book chapter under review, Hill explores similarities and differences in their approaches to mathematics and philosophy. Hill details a less familiar side of Cantor, often thought of as a harbinger of modernity in mathematics.

Cantor was unable to publish his article “Mitteilungen zur Lehre vom Transfiniten” in *Acta Mathematica* in the 1880s. He had to withdraw the article after acceptance, due to concern over its extra-mathematical (and, more specifically, philosophical) content expressed by editor Mittag-Leffler. Already in 1878, Cantor had vowed never to publish in *Crelle's journal*, where his ideas received lukewarm reception from Weierstrass and other heavyweights at Berlin.

Meanwhile, Felix Klein, based at Göttingen, was one of the first to appreciate the originality of Cantor's ideas, and to publish his work in the *Mathematische Annalen*. Cantor eventually published his “Mitteilungen” in a philosophy journal in 1887 [*Z. Phil. Phil. Krit.* **91**, **92** (1887); JFM 19.0044.02]. A related one-page note appeared in the *Mathematische Annalen* in 1889, as noted on page 336, note 29, in J. W. Dauben's book [*Georg Cantor*, Princeton Univ. Press, Princeton, NJ, 1990; MR1082146].

Hill notes that Cantor's book *Mannigfaltigkeitslehre*, published in 1883 [*Grundlagen einer allgemeinen Mannigfaltigkeitslehre (Foundations of a general theory of manifolds)*, Teubner, Leipzig, 1883; JFM 15.0453.01], emphasized the idealist foundations of his theories based on Platonist principles. By “set”, Cantor meant something related to the Platonic *eidos*, or idea.

Cantor spoke of awakening and bringing to consciousness the knowledge, concepts, and numbers slumbering in us. Such ideas have their source in Plato's theory of recollection.

In his “Mitteilungen”, Cantor described his transfinite numbers as a special form of Plato's *arithmoi noetoi* or *eidetikoi*.

Cantor elaborated a theory of abstracting numbers from reality. He considered his technique of abstracting as the only possible foundation for his Platonic conception of numbers. Cantor viewed the abstracting technique as a way of “focusing on pure, abstract arithmetical properties and concepts that would divorce them from any sensory apprehension of the particular characteristics of the objects figuring in the sets and so free mathematics from psychologism, empiricism, Kantianism and insidious appeals to intuitions of space and time.” (p. 173)

Husserl's early philosophical stance was influenced by the empirical psychology of Franz Brentano. At this stage, Husserl endorsed Cantor's position concerning both *abstracting* and *awakening*. Thus, Husserl “commended [Cantor] for having written with a great deal of precision in the ‘Mitteilungen’ that for ‘the formation of the general concept ‘five’ one needs only a set (for example all the fingers of my right hand) which corresponds to this cardinal number; the act of abstraction with respect to both the properties and the order in which I encounter these wholly distinct things, produces or rather awakens the concept ‘five’ in my mind.’” (p. 174)

At a later stage in his development, Husserl became “a committed Platonic idealist

persuaded that pure mathematics was a strictly self-contained system of doctrines to be cultivated by using methods essentially different from those of natural science” (p. 177).

Hill notes that, “although Cantor’s ideas probably helped pry Husserl away from psychologism and turn him in the direction of Platonic idealism, [Husserl] always maintained that Hermann Lotze’s work was responsible for his conscious, radical rejection of psychologism and the accompanying Platonism.” (ibid.)

Hill mentions two objections Husserl may have raised to Cantor’s approach at this stage in Husserl’s development:

- (1) “Husserl might have fairly wondered in just what way the cardinal number belonging to a set is an abstract image in our intellect; or
- (2) “exactly how the act of abstraction awakened the number concepts in Cantor’s mind.” (p. 178)

Hill summarizes the internal contradictions of Cantor’s stance in the following terms:

“In the same passage of the *Mannigfaltigkeitslehre* in which Cantor explicitly rejected the belief that ‘the source of knowledge and certainty is located in the senses or in the so-called form of pure intuition of the world of presentation’, he wrote that ‘certain knowledge . . . can only be obtained through concepts and ideas, which are at best only stimulated by outer experience, but which are principally formed through inner induction, like something which, so to speak, already lay within us and is only awakened and brought to consciousness.’” (pp. 178–179)

On the subject of Cantor’s metaphysics, Hill quotes the following passage from Cantor’s letter to Thomas Esser in 1896:

“[The] grounding of the principles of mathematics and natural science is a matter for metaphysics. Metaphysics has therefore to look upon these two sciences not only as its servants and helpers but also as its children which it should not let out of its sight, but must watch over and control.” (p. 181)

Cantor was convinced that:

“[G]eneral *Mengenlehre* . . . belongs thoroughly to metaphysics. You can easily convince yourself of this by examining the basic concepts of *Mengenlehre*, the categories of cardinal number and ordinal type, and noticing not only the degree of their generality, but also how thinking [*Denken*] with them is fully pure, so that there is not the slightest room for fantasy.” (ibid.)

Dauben provides the following interpretation of what Cantor meant by metaphysics:

“Whenever Cantor spoke of metaphysics he meant the philosophical study of the relations between the constructs of mind and the objects of the external world. Thus the study of the abstract theory of the transfinite numbers was the business of mathematics, but the study of the realization or embodiment of the transfinite numbers in terms of the objects of the phenomenological world was the concern of metaphysics. And so metaphysics assumed its place in Cantor’s continuing program to establish the legitimacy of his new theory.” [J. W. Dauben, op. cit. (p. 125)]

Hill notes that Cantor’s metaphysics on occasion crossed the line into mysticism. She quotes the following passage from Cantor:

“[T]he whole numbers both separately and in their actual infinite totality exist in that highest kind of reality, etc.” (p. 181)

Given such beliefs concerning numbers in reality, it is not surprising that resolving the continuum hypothesis, necessarily an aspect of such reality, became a lifelong obsession for Cantor.

With regard to Cantor’s mysticism, Hill comments: “Such considerations may explain why Husserl wrote in the *Logical investigations* of banishing ‘all metaphysical fog and all mysticism’ from mathematical investigations into numbers and manifolds like those

of Cantor” (p. 182).

Hill addresses the matter of Frege’s criticisms of Cantor and Husserl. She notes that some of Frege’s criticisms of Cantor were later echoed in Frege’s criticisms of Husserl. Frege’s particular target was the technique of abstracting. Thus, Hill notes:

“When Frege went on to charge that Husserl had taken ‘the road of magic rather than of science,’ we have a good clue as to whom else he wished to attack, for in reviewing the ‘Mitteilungen’, he had accused Cantor of the very same thing. He called the verb ‘[to] abstract’ a psychological expression to be avoided in mathematics.” (p. 184)

Hill notes a further connection between Frege’s critiques of Cantor and Husserl:

“Cantor’s presence is again felt when Frege wrote that according to Husserl ‘numbers are supposed to be *Vorstellungen*, the results of mental processes or activities’” (p. 185).

Hill notes that “Husserl quickly came to judge his first attempts to clarify the true meaning of the fundamental concepts of the theory of sets and cardinal numbers to have been a failure. He not only began to have doubts about psychological analyses of sets, but he expressed doubts about set theory itself. He confessed to having been disturbed, and even tormented, by doubts about sets right from the very beginning. He specifically put Cantorian sets, ‘the *Mannigfaltigkeitslehre* in the broadest sense’, into the category of pure logic that was a source of torment to him.” (p. 186)

In summary, at an early stage in Husserl’s career, “the naive epistemological theorizing in which Cantor was so earnestly engaging while Husserl was grappling with analogous questions could well have seemed to Husserl to be amenable to clarification through Brentano’s teachings. . . . Cantor’s technique for extracting numbers from reality through abstraction might be a psychological process” (p. 192).

At a later stage, “it was Lotze’s work that was responsible for his conscious, radical rejection of psychologism, his espousal of Platonism and newfound comprehension of Bolzano’s work on pure logic and, therefore, for his adoption of metaphysical and epistemological views that Brentano had taught him to consider odious and despicable” (p. 193).

It is to be regretted that J. N. Mohanty’s influential volume on Husserl [*The philosophy of Edmund Husserl: a historical development*, Yale Univ. Press, New Haven, CT, 2008] is not mentioned either in Hill’s text or for that matter anywhere else in the book of which Hill’s text is a chapter.

The article often employs the term *manifold* in the sense of *multitude*, which may be confusing to a mathematically trained English reader.

{For the collection containing this paper see [MR3837155](#)}

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