

Projective geometry 88-524-01 sample problems

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source: Fishback, W.: Projective and Euclidean geometry.

page 53-54.

1. carry out the construction of a line d such that a, b, c, d is harmonic, for given a, b, c .
2. assume $H(A, B, C, D)$. Find seven other orders of the points A, B, C, D for which the harmonic relation holds.
recall $ABCD = CDAB = BADC = DCBA = x$
 $R(BACD) = 1/x$, $R(ACBD) = 1-x$, $R(ADBC) = (x-1)/x$.
3. Prove that if C is the midpoint of AB then the fourth harmonic point D , so that $H(ABCD)$, is an ideal point.
4. Assume a line c bisects an angle determined by a and b . Let d is the fourth harmonic line so that $H(abcd)$. Find the angle between c and d .

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5. Assume that $H(ABCD)$. Let T be a projectivity such that $T(ABC) = ABD$. Prove that $T(D) = C$.

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6. The affine (x, y) plane is completed to a projective plane using homogeneous coordinates where $x = x_1/x_3$ and $y = x_2/x_3$. Find coordinates of the ideal point or points of each of the following lines:
 - (a) $2x_1 - 3x_2 + 4x_3 = 0$;
 - (b) $7x_1 + 2x_2 + 3x_3 = 0$;
 - (c) $x_1 = 0$;
 - (d) $x_2 = 0$;
 - (e) $x_3 = 0$.
7. Find the equation in homogeneous coordinates of each of the following lines:
 - (a) the x -axis;
 - (b) the ideal line;
 - (c) the line through $[3, 7, 1]$ and $[2, 3, 1]$;
 - (d) the line through $[3, 7, 1]$ and $[2, 3, 0]$.