HW 1 - Analytic and Differential geometry 88-201

Submission deadline: March 27, 2025.

1. Write the following summations given in Einstein notation in full form.

   (a) $a^i{}_j b^j{}_k c^k_\ell$

   (b) $a_{ij} v^i v^j$

   (c) $\delta_{ij} a^{ij}$

   where $i, j, k, \ell \in \{1, 2, 3\}$ and δ is the Kronecker delta function.

2. Calculate the following expression given in Einstein summation notation:

$$\delta^i{}_j \delta^j{}_k \delta^k_i$$

   where $i, j, k \in \{1, 2, \cdots, n\}$. Write which indices are summation indices and which are free indices.

3. Let $a, b, c, d \in \mathbb{R}^3$, prove the following identities:

   (a) $a \times (b \times c) = \langle a, c \rangle b - \langle a, b \rangle c$

   (b) $\langle a \times b, c \times d \rangle = \langle a, c \rangle \langle b, d \rangle - \langle a, d \rangle \langle b, c \rangle$