HW 4 - Analytic and Differential geometry 88-201

Submission deadline: May 5, 2025.

1. Compute the curvature of the following curves, and if possible, state the points where the curvature is maximal:

   (a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with $0 < a < b$

   (b) $x^3 - y = 2$

2. Find unit speed parametrization for the following curves:

   (a) $\alpha(t) = (1 + 2\cos t,\ -3 + 2\sin t)$

   (b) $\alpha(t) = \left(t,\ \frac{1}{3}\sqrt{(2+t^2)^3}\right)$

   (c) For $a > 0$, $\alpha(t) = \left(t,\ a\cosh\left(\frac{t}{a}\right)\right)$

3. Compute the curvature of the following curves. Simplify as much as possible:

   (a) $\alpha(t) = (1 + 2\cos t,\ -3 + 2\sin t)$

   (b) $\alpha(t) = \left(t,\ a\cosh\left(\frac{t}{a}\right)\right)$

4. Let $\alpha : [0, L] \to \mathbb{R}^2$ be a regular closed curve given in unit speed parametrization.

   Prove that if the curvature $k(s)$ is monotonic, then it is constant.