HW 5 - Analytic and Differential geometry 88-201

Submission deadline: May 15, 2025.

1. Compute the fundamental forms of the following surfaces:

   (a) A surface given implicitly by $z = f(x, y)$.

   (b) $X(\theta, \phi) = (\cosh\phi\cos\theta, \cosh\phi\sin\theta, \phi)$

   (c) $X(\theta, \phi) = (\cosh\phi\cos\theta, \cosh\phi\sin\theta, \sinh\phi)$

2. Let $\alpha : [a, b] \to \mathbb{R}^2$ be a simple (non-self-intersecting) and regular curve. Define the cylinder over α as follows:

$$\Phi : (a, b) \times \mathbb{R} \to \mathbb{R}^3$$

$$\Phi(t, u) = (\alpha^1(t), \alpha^2(t), u)$$

   where $\alpha = (\alpha^1, \alpha^2)$ Show that there exist coordinates on the cylinder (a parameterization of α) such that $(g_{ij}) = I$.

3. The unit sphere is defined by the following parameterization:

$$X(\theta, \phi) = (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi)$$

   where $(\theta, \phi) \in [0, 2\pi] \times [0, \pi] = U$. We consider the curve α defined by:

$$\alpha(t) = (\pi, 2t), \quad t \in \left[0, \frac{\pi}{2}\right]$$

   (a) Find the length of the curve α.

   (b) Find the length of the curve $X \circ \alpha$.