

HW 7 - Analytic and Differential geometry 88-201

Submission deadline: June 16, 2025.

1. Compute the second fundamental form coefficients L_{ij} and the coefficients L^i_j of Weingarten map for the following surfaces:

(a) Cone:

$$X(\theta, \phi) = (\phi \cos \theta, \phi \sin \theta, k\phi), \quad k > 0$$

(b) Helicoid:

$$X(u, v) = (u \cos v, u \sin v, kv), \quad k > 0$$

2. Let M be a surface given by the equation $z = f(x, y)$. Assume $f(0, 0) = 0$, and the tangent plane to the surface at the point $(0, 0, 0)$ is the xy -plane:

$$T_0(M) = \{(x, y, 0) : x, y \in \mathbb{R}\}$$

(a) Compute the coefficients L_{ij} of the second fundamental form at the point $(0, 0, 0)$.

(b) Compute the Gaussian curvature at the origin.

3. Express the following quantities in terms of Γ^k_{ij} , L_{ij} , and g_{ij} , and simplify as much as possible:

(a) $\langle x_j, x_{pq} \rangle g^{jp}$

(b) $\langle x_{pqr}, n \rangle$

(c) $\langle x_{pq}, n_s \rangle \delta_m^q$

(d) $\langle x_{ij}, n_k \rangle \delta_m^k g^{mj}$