HW 8 - Analytic and Differential geometry 88-201

Submission deadline: July 3, 2025.

You can answer 2 out of 3 questions. (If you answer 3, the 2 with a higher score will count).

1. Prove that the following surfaces are minimal

   (a) $X(u,v) = (u\cos v, u\sin v, v)$

   (b) $X(u,v) = \begin{pmatrix} 2\cos v\sinh u - \frac{2}{3}\cos(3v)\sinh(3u), \\ 2\sin v\sinh u - \frac{2}{3}\sin(3v)\sinh(3u), \\ 2\cos(2v)\cosh(2u) \end{pmatrix}$

2. Consider a surface with coordinates (x,y), with the metric

$$(g_{ij}) = \begin{pmatrix} 1 & 0 \\ 0 & y \end{pmatrix} \quad \text{for } y > 0.$$

   Compute the Gaussian curvature at every point on the surface.

3. Find a parametrization of the sphere as a surface of revolution and compute its Gaussian curvature $K(\phi,\theta)$ in two ways:

   (a) Using the determinant of the Weingarten map.

   (b) Using the formula:

$$K = \frac{2}{g_{11}}(\Gamma^2_{1[1;2]} + \Gamma^j_{1[1}\Gamma^2_{2]j})$$