Solutions to exam in 88-201 moed A july '17

Problem 1. The proof of the theorema egregium can be found in the choveret of the course at http://u.math.biu.ac.il/~katzmik/egreglong.pdf in sections 10.8 and 10.9 on pages 103-105 (the section and page numbers may change in the future as the choveret is edited).

Problem 2. (a) To determine the conic one can either complete the square or diagonalize the matrix of coefficients using eigenvalues and eigenvectors. One then applies theorem 2.7.1 and its corollaries on page 26 of the choveret (the theorem number and page number may change in the future as the choveret is edited).

(b) Once the quadratic form is diagonalized, we recognize the standard equation of a paraboloid as described in example 3.4.1 on page 31 (the numbers may change as the choveret is edited in the future). To find the Gaussian curvature we first prove that the point is a critical point, and then apply Theorem 3.5.7 on page 34 to compute the curvature via the Hessian.

Problem 3. Find the maximum of the curvature of a curve as in choveret section 4.5 page 42 (page numbers may change in the future as the choveret is edited).

Problem 4. (a) The formula relating the Laplacian to the mean curvature in isothermal coordinates is proved in choveret Section 10.3 on pages 97-98 (the numbers may change in the future as the choveret is edited).

(b) The minimality of the catenoid is proved in choveret Example 10.4.4 on page 100 (the numbers may change as the choveret is edited in the future).

(c) The minimality of the Scherk surface is proved in choveret Theorem 10.2.2 on pages 96-97 (the numbers may change as the choveret is edited in the future).

Problem 5. Numerous examples of simplification of formulas in Einstein index notation were treated both in class and the exams from previous years. Make sure also to classify the indices as being either free indices or summation indices.

Problem 6. Parametrizing the curve by $\alpha(t)$ we note that at the point closest to the origin one has $\langle \alpha(t), \alpha(t) \rangle' = 0$ by calculus. Applying Leibniz rule we obtain $\langle \alpha'(t), \alpha(t) \rangle = 0$. Therefore the tangent vector $\alpha'(t)$ is orthogonal to the position vector $\alpha(t)$.