Sergeyev, Yaroslav D.
Numerical infinities and infinitesimals: methodology, applications, and repercussions on two Hilbert problems. (English) Zbl 06856857
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In this paper Sergeyev has generalized a form of elementary algebra to a `calculus of infinity'. An exemplar of this algebra is the identity

$$1 + x + x^2 + \cdots + x^N = \frac{x^{N+1} - 1}{x - 1}.$$

Here the summation on the left has its usual meaning and N represents any positive integer. Sergeyev's idea is to take the left-hand side as an "infinite series" and replace N by a new "infinite quantity" G called the "grossone". He would then write

$$1 + x + x^2 + \cdots + x^G = \frac{x^{G+1} - 1}{x - 1}.$$

There has been controversy about Sergeyev's work and its applications. It is the contention of the reviewer that the Sergeyev grossone, G, can simply be interpreted as a generic large natural number and that there is no new theory of infinity involved. Part of the controversy stems from the fact that Sergeyev does not make an attempt to give a logical foundation for his new idealized infinite and infinitesimal elements. For example, we can write

$$1 + 2 + 2^2 + \cdots + 2^N = \frac{2^{N+1} - 1}{2 - 1} = 2^{N+1} - 1$$

For this we see that we have corrected a paradoxical identity

$$1 + 2 + 2^2 + \cdots = -1$$

to a correct identity. It is correct for any finite value of N. Similarly,

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^N} = \frac{(1/2)^{N+1} - 1}{-1/2} = -\frac{1}{2^N} + 2$$

and here we see the limit where, as N goes to infinity (in the usual language), the series goes to 2 and we have the non-paradoxical limit

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 2.$$

And one more example,

$$1 + (-1) + (-1)^2 + (-1)^3 \cdots + (-1)^N = \frac{(-1)^{N+1} - 1}{-2} = -\frac{1}{2}(-1)^{N+1} + \frac{1}{2}.$$

Here for odd N the value is zero and for even N the value is 1 and these are the partial sums of the corresponding series. Sergeyev would write

$$1 + (-1) + (-1)^2 + (-1)^3 + \cdots + (-1)^G = \frac{(-1)^{G+1} - 1}{-2} = -\frac{1}{2}(-1)^{G+1} + \frac{1}{2}.$$

For him the "quantity" $(-1)^{G+1}$ is a new number generated from his grossone G. One can go on in this fashion, tracking the correct algebraic results of using a generic N. Thus for example $1/2^N$ can be seen as a generic small number, and $1/2^G$ is an infinitesimal. This way of thinking about infinitesimals was part of calculus at the beginning of the twentieth century. In the words of Granville, "an infinitesimal is a non-zero variable whose limit is zero". This idea is of course part of the modern logical formulation of infinitesimals. Sergeyev has provided, by rhetoric and many examples, a permission to go back to this way of thinking about infinite numbers and infinitesimals in the context of generic elementary algebra. The paper is concrete and replete with many examples of the use of calculations of this kind. Most of the calculations can be understood by interpreting the grossone G as a very large finite number, but there are interesting issues about infinity under the surface. For example, consider the sequence $1, 2, 3, \dots, G$. In Sergeyev's way of thinking this is an infinite sequence different from the usual notion of the infinite sequence $1, 2, 3, \dots$. The difference is that the Sergeyev sequence has an infinite number G at its top and one can count down from G as well as up to G. That is, we can consider $G, G-1, G-2, G-3, \dots$ and this descending sequence does not end. This shows that the infinite numbers of Sergeyev are distinct from the well-ordered transfinite ordinals. They are of intuitive interest and can be compared with infinite numbers in existing extended real number systems such as those produced by Abraham Robinson and John Horton Conway. Sergeyev has forged an original path to these infinities, infinitesimals and their calculus. He shows how to use his ideas by many examples. However, he has not attempted to write a logical foundation for his ideas. It is in this lack of foundations that the paper's faults arise. In the introduction to his paper, Sergeyev writes, "In particular, it is shown that the new approach allows one to observe mathematical objects involved in the Hypotheses of Continuum and the Riemann zeta function with a higher accuracy than it is done by traditional tools. It is stressed that the hardness of both problems is not related to their nature but is a consequence of the weakness of traditional numeral systems used to study them. It is shown that the introduced methodology and numeral system change our perception of the mathematical objects studied in the two problems." Nothing of the sort is actually available here. There is certainly no new insight given here into either Cantor's Continuum Problem or the Riemann Hypothesis. There is a psychological problem that will vanish eventually. At first one is puzzled. Then it looks familiar -- generic elementary algebra. But this author is using a different language. Maybe there is something new. In this reviewer's opinion there is nothing new in this work except a notation G that represents a large number. Editorial Comment: From the statement of the editorial board: It was a serious mistake to accept it for publication. Owing to an unfortunate error, the entire processing of the paper, including the decision to accept it, took place without the editorial board being aware of what was happening. The editorial board unanimously dissociates itself from this decision.

Reviewer: Louis H. Kauffman (Chicago)

MSC:

03H05 Nonstandard models in mathematics

68Q99 Theory of computing

MathOverflow Questions:

Cargo Cult Science in mathematics?

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