May 13, 2020

Differential geometry 88-826 homework set 4

1. Recall that the n-th exterior power $\Lambda^n(\mathbb{R}^n)$ of $\mathbb{R}^n$ is spanned by the single element $\omega = e_1 \wedge e_2 \wedge \cdots \wedge e_n$.

(a) Consider the 2-multivector $\alpha = e_1 \wedge e_2 + e_3 \wedge e_4 \in \Lambda^2(\mathbb{R}^4)$. Express the product $\alpha \wedge \alpha$ explicitly as a multiple of of $\omega \in \mathbb{R}^4$.

(b) Consider the 2-multivector $\alpha = e_1 \wedge e_2 + e_3 \wedge e_4 + e_5 \wedge e_6$ in $\Lambda^2(\mathbb{R}^6)$. Express the product $\alpha \wedge \alpha \wedge \alpha$ explicitly as a multiple of $\omega \in \mathbb{R}^6$.

2. Consider the differential 2-form $\eta = f(u^1, \ldots, u^n)du \wedge dv$, where du and dv are among the coordinate forms du^i. Prove that the 4-form $dd\eta$ identically vanishes.

3. Consider the Eisenstein lattice $L_E \subseteq \mathbb{C}$ spanned by the cube roots of unity. Prove that $\lambda_1(L_E^*)\lambda_1(L_E) = \frac{2}{\sqrt{3}}$.