

May 24, 2023

DIFFERENTIAL GEOMETRY 88-826 HOMEWORK SET 4

Due Date: 7 june '23

1. Let $r > 0$ and let D be the *unbounded* region

$$D = \{(x, y) \in \mathbb{R}^2: x^2 + y^2 \geq r^2\}$$

endowed with the standard orientation $dx \wedge dy$. Determine the induced orientation on ∂D and compare it to $d\theta$.

2. Let M be a 6-dimensional manifold with $b_2(M) = 1$, with an integer de Rham class $\omega \in L_{\text{dR}}^2(M)$ such that $\omega^{\cup 3}$ is the fundamental cohomology class of M . Find the supremum over all Riemannian metrics g on M of the ratio $\frac{\text{stsys}_2(g)}{\sqrt[3]{\text{vol}(g)}}$, with proof.

3. Let $M = \mathbb{C}\mathbb{P}^1 \times \mathbb{C}\mathbb{P}^2 \times \mathbb{C}\mathbb{P}^3$. Prove that all metrics g of volume 1 on M satisfy $\text{stsys}_2(g) \leq C_n$ for a suitable constant C_n independent of the metric.

4. Determine which of the following 8-dimensional manifolds satisfy a stable systolic inequality for stsys_2 with a constant independent of the metric:

- (1) $S^2 \times S^8$;
- (2) $S^2 \times \mathbb{C}\mathbb{P}^3$;
- (3) $S^2 \times S^2 \times S^4$;
- (4) $S^2 \times S^2 \times \mathbb{C}\mathbb{P}^2$;
- (5) $\mathbb{C}\mathbb{P}^2 \times S^4$;
- (6) $\mathbb{C}\mathbb{P}^3 \times T^2$.