88-826 DIFFERENTIAL GEOMETRY, MOED A
28 JUL '10

Duration of the exam: $2\frac{1}{2}$ hours.

All answers must be justified by providing complete proofs.

1. The lattice L_E of Eisenstein integers is the lattice in $\mathbb{C} = \mathbb{R}^2$ spanned by the cube roots of unity. Find the dual lattice L_E^* to the lattice L_E and compute the first successive minimum $\lambda_1(L_E^*)$.

2. The cotangent plane T_p^* at a nonzero point p of the Euclidean plane $\mathbb{R}^2$ in polar coordinates (r, θ) has a basis of 1-forms dr and $d\theta$, while the tangent plane T_p has a basis $\frac{\partial}{\partial r}, \frac{\partial}{\partial \theta}$. Find an orthonormal basis for T_p^* and an orthonormal basis for T_p.

3. Let M be the torus in $\mathbb{R}^3$ obtained by rotating the closed curve in the (x, z)-plane defined by the equation $(x-4)^2 + z^2 = 4$ around the z-axis. Consider the tangent bundle (eged hameshik) of the torus, denoted TM. Is TM diffeomorphic to $M \times \mathbb{R}^2$?

4. Let V be a finite dimensional real vector space equipped with an real inner product $\langle\ ,\ \rangle$. Construct a natural isomorphism between V and its dual V^*.

5. Let e_1, e_2, e_3, e_4 be the standard basis for $\mathbb{R}^4$. Given a 2-form $A_{i,j,k,\ell} \in \Lambda^2(\mathbb{R}^4)$ defined by the formula

$$A = e_i \wedge e_j + e_k \wedge e_\ell, \qquad (0.1)$$

determine when A is decomposable (simple) and when it is not, as a function of the indices i, j, k, ℓ.

6. On the punctured complex plane $P = \mathbb{C} \setminus \{0\}$, consider the 1-form $d\theta$ where θ is the argument of $z \in P$. Is $d\theta$ in the image of the homomorphism $d : C^\infty(P) \to \Omega^1(P)$ (the exterior derivative)?

7. Calculate the comass $\|\alpha\|$ of the symplectic form α on $\mathbb{C}^\mu$.

GOOD LUCK!