Differential geometry 88-526-01 homework set 3
November 9, 2010

1. Consider the parabola

$$x = z^2 + \frac{1}{4}$$

in the xz-plane. Let M be the surface of revolution obtained by rotating the parabola around the z-axis.

(a) find a parametrisation of the parabola;
(b) find a parametrisation of M;
(c) find the ratio $\frac{\kappa_1}{\kappa_2}$ of the principal curvatures of M.

2. Let $x(u^1, u^2)$ be a parametrized surface in $\mathbb{R}^3$. Find an expression for the scalar product $\langle x_{ij}, x_{kl} \rangle$ in terms of a combination of the following data: the Christoffel symbols, the coefficients of the first fundamental form, and the coefficients of the second fundamental form.

3. Let C be the curve in the (x, z)-plane which is the locus of the equation $(x-3)^2 + z^2 = 1$.

(a) Find a unit speed parametrisation of C;
(b) find a parametrisation $x(\theta, \phi)$ of the surface of revolution $M \subset \mathbb{R}^3$ obtained by rotating C around the z-axis;
(c) calculate the coefficients of the first and second fundamental forms;
(d) calculate the Gaussian curvature $K(\theta, \phi)$;
(e) determine when the Gaussian curvature is positive;
(f) describe geometrically the region on M where the Gaussian curvature is positive.

4. This problem concerns the calculation of Gaussian curvature K.

(a) Describe at least three possible ways of calculating K.
(b) Which of the approaches in (a) are applicable if the data one is given is that the metric is defined in coordinates (u^1, u^2) by the metric coefficients $g_{ij}(u^1, u^2) = \frac{1}{(u^2)^2}\delta_{ij}$?
(c) Calculate K for the metric in (b).