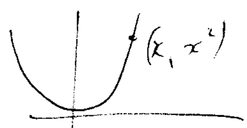
(1) $\boxed{\text{Recall } \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}}$ [illegible] 

$\boxed{\text{EO 4 / 62}}$ [illegible]

88-136
Calculus 89-118. 20 feb

Recall function represented by its graph: pairs $(x, f(x))$ in $\mathbb{R}^2$. ~~Form~~

In general, Def. A real function of one variable is a set f of ordered pairs, (זוגות סדורים)

Sometimes undefined. $f(x) = \frac{1}{x}$ undefined for $x = 0$

such that for every real number a, one of the following two things happens:

(i) ~~either~~ there is exactly one real number b for which the ordered pair (a, b) is a member of f. In this case we say that $f(a)$ is defined, and write $f(a) = b$. The number b is called the value of f at a.

(ii) there is no real number b for which the ordered pair (a, b) is a member of f. In such case we say that $f(a)$ is undefined.

Thus, writing $f(a) = b$ means that the ordered pair (a, b) is an element of f

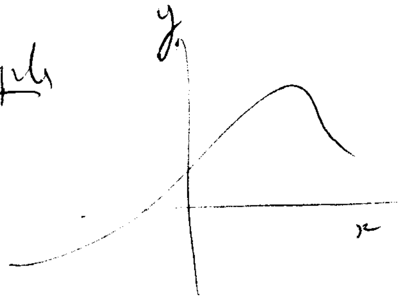
Graphing The graph of a real function f is the set of points $P(x, y)$ in ~~$\mathbb{R}$~~ the ~~xy~~ plane such that $y = f(x)$.

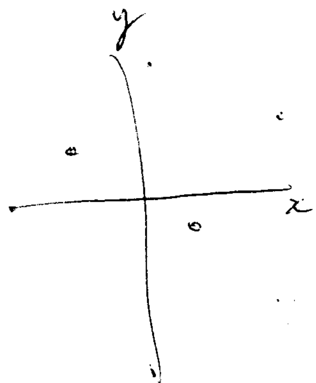
A set of points in the plane is the graph of some function iff for every vertical line (ישר אנכי) one of the fol. things happen:

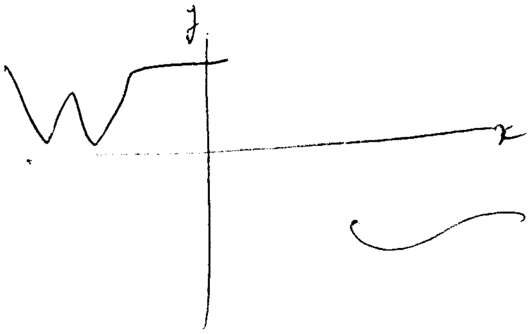
(i) ~~either~~ exactly one point of the line belongs to the set

(ii) no point on the line belongs to the set.

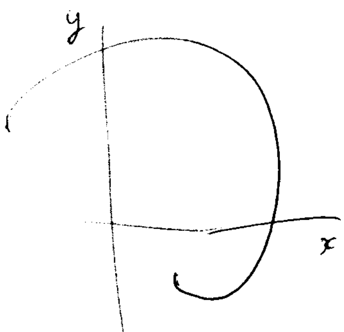
Examples

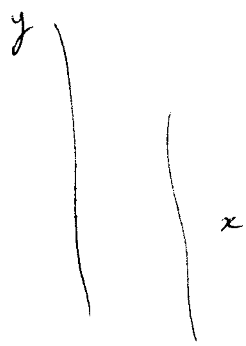


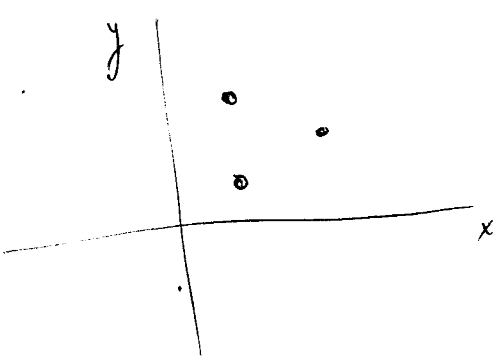


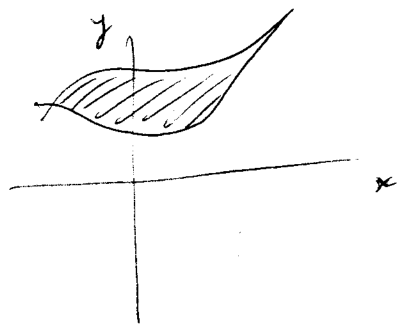


Example not a graph:







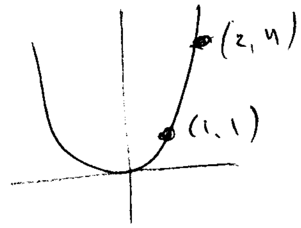


Ex The Square function:

$$f(x) = x^2$$

$f(0) = 0$
$f(2) = 4$
$f(-3) = 9$

$f(r) = r^2$
$f(r+1) = r^2 + 2r + 1$.



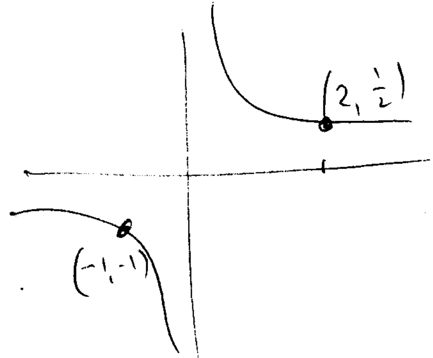
Graph is the parabola

Ex. The reciprocal function

$$g(x) = \frac{1}{x}$$

Defined for all nonzero x. Undefined at $x = 0$.

$g(0) =$ undefined
$g(2) = \frac{1}{2}$
$g\left(-\frac{1}{3}\right) = -3$
$g(r) = \frac{1}{r}$
$g(r+1) = \frac{1}{r+1}$



The graph has equation $y = \frac{1}{x}$

alternatively, $xy = 1$.

Important to distinguish between symbol f and expression $f(x)$.

f function (סמן פונקציה)

$f(x)$ term / ביטוי ~~([illegible])~~

<u>Example.</u> Let h be the function given by the rule

$$h(t) = t^3 + 1$$

t variable

h function

$h(t)$ is a term

The following expressions are also terms:

$$h(\tfrac{1}{2}),\quad h(x),\quad h(t^3),\quad h(t^3+1),$$

$$h(x) - h(t),$$

$$h(t+\Delta t)$$

$$h(t+\Delta t) - h(t)$$

Calculate them:

$\vdots$

$$h(t+\Delta t) - h(t) = \dots = 3t^2\,\Delta t + 3t\,\Delta t^2 + \Delta t^3.$$

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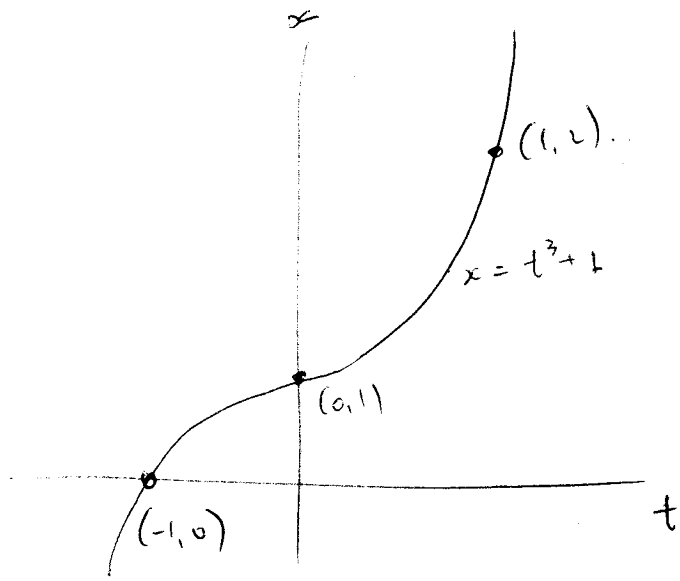
graph



Def. **Domain** set of numbers s.t $f(x)$ is defined.
תחום

Range of f is set of all values $f(x)$, when x is in the domain of f
תמונה ~~תמונה~~ ~~טווח~~

Ex 1. $f(x) = x^2$. domain $= \mathbb{R}$
range $= [0, \infty)$ nonneg. reals.

Ex 2 reciprocal function:
both domain and range are all real numbers s.t. $x \neq 0$.

Function h given by rule

Ex. $h(t) = t^3 + 1$ has domain $= \mathbb{R}$
range $= \mathbb{R}$

Ex. Let f be function given by rule

$$f(x) = \sqrt{1-x^2} \quad (\text{pos. sq. root})$$

Domain of f is interval $[-1, 1]$.

Range of f is $[0, 1]$.

E.g. $f(-2)$ undefined.

$f(-1) = 0$

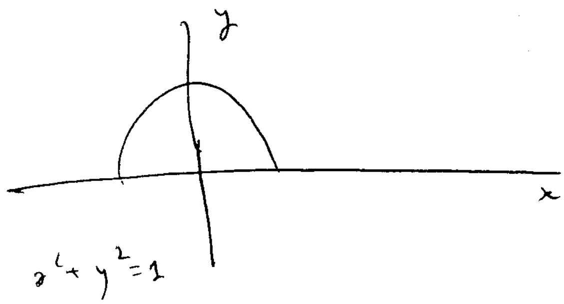
$f(0) = 1$, $f\left(\frac{1}{2}\right) = \sqrt{\frac{3}{4}}$

$f(2)$ undefined.

Graph: equation $y = \sqrt{1-x^2}$

Also can be written as

$$x^2 + y^2 = 1, \quad y \geq 0.$$



$x^2 + y^2 = 1$
$y \geq 0$

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Ex Constant function

Let c real number.

Function f given by rule

$$f(x) = c$$

is called constant function with value c.

domain: $\mathbb{R}$

range: $\{c\}$

Ex. constant function with value 5:

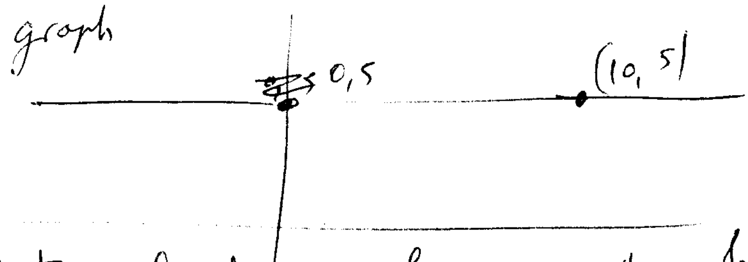
$$f(x) = 5$$

$$f(0) = 5$$

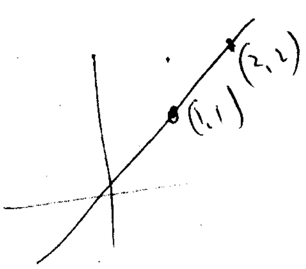
$$f(-3) = 5$$

$$f(1000000) = 5$$

graph



Ex Identity function f is given by rule



$$f(x) = x$$

Absolute value function

Definition The abs val fcn $|\ |$ is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

e.g. $|3| = 3$

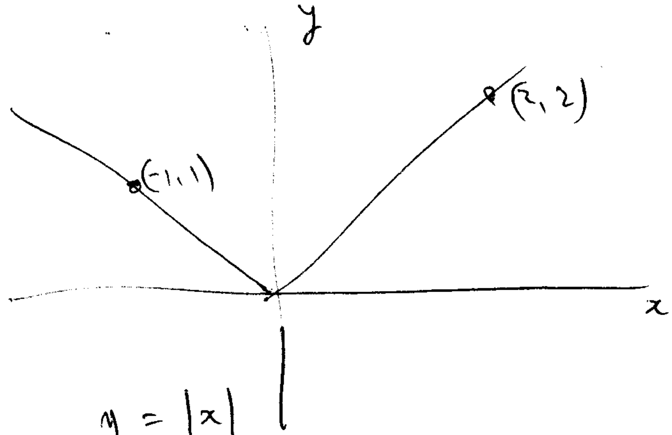
$|0| = 0$

$|-3| = 3$

Domain : $\mathbb{R}$

Range : $[0, \infty)$

Formula $|x| = \sqrt{x^2}$

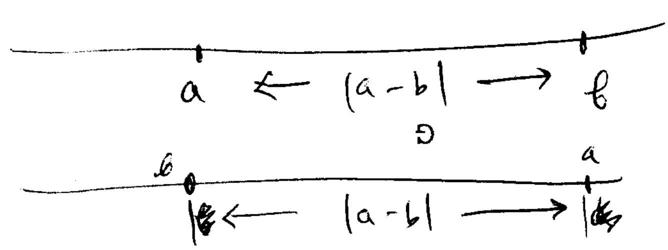


$y = |x|$

Similarly, let a, b two points on real line Then

$$|a-b| = \begin{cases} a-b & \text{if} \\ b-a & \text{if} \end{cases}$$

distances



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Theorem. Let a and b be real numbers. Then

(i) $|-a| = |a|$

(ii) $|ab| = |a|\,|b|$

(iii) If $b \neq 0$ then $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Proof Use equation $|x| = \sqrt{x^2}$.

Then (i) $|-a| = \sqrt{(-a)^2} = \sqrt{a^2} = |a|$

etc.

NB. false equation $|a+b| = |a| + |b|$.

e.g. $|2+(-3)| = 1$

while $|2| + |-3| = 5$.

Functions in nature:

geometry: πr^2 = area of circle

$4\pi r^2$ = surface area of sphere

$\frac{4}{3}\pi r^3$ = volume

$\sin\theta$ = sine of angle θ.

Physics: $s(t)$ = distance a particle travels ~~of~~ from time 0 to time t.

$v(t)$ = velocity of particle at time t
מהירות

$a(t)$ = acceleration at time t
תאוצה

Definition. A real function f of two variables is a set of ordered triples of real numbers s.t. for every ordered pair (a, b), of f the foll 2 things occurs.

(i) There is exactly one ~~ordered~~ real number c such that the ordered triple (a, b, c) is a member of f. In this case, we write

$$f(a, b) = c$$

(ii) There is no real number such that the triple ...

Then $f(a, b)$ called undefined.

domain: set of ordered pairs

Examples. Sum $f(x, y) = x + y$

product $f(x, y) = xy$

חיסור difference $f(x, y) = x - y$

חילוק quotient $f(x, y) = \frac{x}{y}$ ← domain pairs (x, y) st $y \neq 0$

Application geometry:

ab = area of rectangle (מלבן) sides a, b

abc = volume rect. solid (תיבה מלבנית)

$\frac{1}{2} bh$ = area triangle base b height h

$\pi r^2 h$ = vol cylinder with circular base radius r height h

$\sqrt{x^2 + y^2}$ = distance from origin (ראשית) to (x, y).