NOTES FROM THE UNDERGROUND

ABSTRACT. We present a critique of the prevailing historical interpretation of Leibnizian calculus, highlighting resistance within the academic community to alternative views on the history of infinitesimals.

In a series of over 80 articles, our group proposed a new paradigm for the history of mathematics over the past 350 years. As may have been anticipated by Thomas Kuhn and Imre Lakatos, our proposal encountered fierce opposition from some of the established historians and philosophers of mathematics. A fresh PhD sympathetic to our proposal appears to have been canceled. A Leibniz scholar felt it so imperative to join a broadside against our work that he contradicted his own earlier Leibniz scholarship. Historians of the previous generation who did not fit the current dogma are either ridiculed or, when this is impossible, are given a far-fetched interpretation to force such a fit. But little by little, the truth is coming out.

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1. Introduction

Over the past 16 years or so, our group has been publishing articles on the history, mathematics, and philosophy of infinitesimals. We have over 80 articles in journals ranging from *Erkenntnis* to the *British Journal for the History of Mathematics*, and from the *Journal of Symbolic Logic* to *The Mathematical Intelligencer*. These can be consulted at `https://u.cs.biu.ac.il/~katzmik/infinitesimals.html`

Doing both mathematics and the history or philosophy of mathematics is a human activity and as such is subject to opinions and prejudices of their practitioners. Originally the hope was that professional historians of mathematics would take an interest in our investigations and/or engage in meaningful debate. Even though our investigations challenge the received narrative of the past 350 years of the history of mathematics, the hope was for some meaningful feedback based on the fact that we have been publishing in a number of leading professional journals.

The feedback started coming in the past few years, and it has been mainly scathing (with some notable exceptions). We have faced institutional and academic resistance to our interpretation of the history of infinitesimals, including attempts to discredit our work. The inescapable feeling about what is going on is that it is a standard example of dogmatic resistance to change and the kind of social activism that seeks above all to protect the old boys' network. This brought with it the realisation that some historians and philosophers are less interested in the truth than in propping up a party line.

One major challenge to the received narrative concerns the status of the Leibnizian calculus. What appears to be currently the received dogma is the so-called syncategorematic interpretation of Leibnizian infinitesimals. This interpretation amounts to the claim that the term *infinitesimal* does not refer. In plain language, the claim means that whenever Leibniz used the term, he meant it as a kind of stenographic shorthand, or placeholder, for a proof by exhaustion using purely Archimedean terms. Such a stenographic narrative appeared in the second, 1990 edition of Ishiguro's book [24] on Leibniz.

Such a narrative has been refuted many times over, originally by Leibniz himself in a pair of 1695 texts [43], [44]. Here Leibniz made it clear that his 'incomparable' infinitesimals violate the property formulated in Euclid, Book V, Definition 4 (in the edition Leibniz was using, the property appears as Definition 5) when compared to 1. Therefore the term *infinitesimal* does refer. Namely, it refers to a fictional mathematical entity.

The definition is usually translated as follows:

> Magnitudes are said to have a ratio to one another which can, when multiplied, exceed one another. (Euclid, Book V, Definition 4)

For a discussion of incomparability see Section 5.3. The property expressed in Definition 4 is closely related to what is known today as the Archimedean property. If Leibnizian infinitesimals violate the Archimedean property, they cannot be merely a placeholder for an exhaustion argument in a purely Archimedean context.

Furthermore, while Leibniz occasionally mentions a 'syncategorematic infinity', he never speaks of 'syncategorematic infinitesimals' in any of his texts available thus far.

Possible reasons why some historians and philosophers would cling to such a deficient stenographic reading are explored in Section 5.8. But propping up the stenographic narrative (a.k.a. the syncategorematic interpretation) was apparently so important that one of the authors of a recent broadside against our work (see Section 4.1) actually contradicted what he wrote in his own publications on this issue; see Section 5.1 on Jesseph.

The status of infinitesimals in the work of Fermat, Euler, Cauchy and others is hotly disputed, as well; see e.g., [35], [9], [5], [33].

2. Laugwitz

Detlef Laugwitz (1932–2000) was a mathematician and historian of mathematics. He published what became the standard biography of Bernhard Riemann in 1996. He authored a series of publications in professional history of science and history of mathematics journals on both Euler and Cauchy; see [39] and [40]. Laugwitz wrote:

> In the realm of the finite it does not really matter mathematically whether we consider the natural numbers as *ordinals* or as *cardinals* or as constituents of an algebraic structure. Cantor had succeeded in showing, by different laws holding for transfinite ordinals and cardinals, that the infinite could clarify the conceptual distinctions. To

> Stolz, Veronese and Levi-Cività we owe early insights in ordered mathematical structures as a *third aspect* of the number concept. This approach prepared the ground for twentieth century developments. (Laugwitz [42, p. 103]; emphasis added)

Such a 'third aspect of the number concept' is what we referred to as a *ringinal* in [30] and [29]. A ringinal can be defined as a number greater than any naive counting number. The inverse of a ringinal is infinitesimal.

Centuries before Levi-Civita (mentioned by Laugwitz), such entities were already in Leibniz; more specifically, in his 1676 manuscript *De Quadratura Arithmetica*, in the Scholium following Proposition 11. Here Leibniz introduces a distinction between *linea infinita terminata* and *linea infinita interminata* (literally: 'bounded infinite lines' and 'unbounded infinite lines'). Traditional mathematical training tends to foster a belief that there are only two legitimate forms of infinite number: *cardinal* and *ordinal*. Following Laugwitz, we will explore a trichotomy of

(1) *cardinal*,
(2) *ordinal*, and
(3) *ringinal*.

The Leibnizian *infinitum terminatum* (see Section 3) behaves as a *ringinal*.

The *infinitum interminatum* is contradictory if taken as an infinite whole, and can only be made sense of syncategorematically (via Aristotelian potential infinity); see [57].

3. *De Quadratura Arithmetica*

Richard Arthur and David Rabouin are among of the signatories of a recent broadside against our work (see Section 4.1). The most comical aspect of the Arthur–Rabouin production is their blatant cherry-picking when it comes to Leibniz's 1676 manuscript *De Quadratura Arithmetica* (DQA). It is instructive to compare their treatment of Proposition 6 and Proposition 11 of DQA.

While commenting on Proposition 6, which they claim to be a triumph of the Archimedean method, they defend DQA fiercely against what they perceive to be insufficient appreciation by Jesseph and others. But consider DQA's Proposition 11 which develops the dichotomy of

(1) *infinitum terminatum* (literally: bounded infinity) vs
(2) *infinitum interminatum* (literally: unbounded infinity).

Leibniz emphasizes that the *interminatum* is contradictory whereas the *terminatum* is not and therefore useful in geometry. In modern terms, Leibniz's unbounded infinity can be exemplified by an infinite line we take for granted today. Meanwhile, bounded infinity is exemplified, in modern terms, by an interval of length H, where H is a nonstandard integer as formalized in Abraham Robinson's nonstandard analysis.

When it comes to Proposition 11, the Arthur–Rabouin team keeps mostly silent, apart from some far-fetched scholasticism bordering on the mystical. In fact what Leibniz wrote in his Scholium to Proposition 11 directly refutes their central thesis concerning an alleged contradictory nature of *infinita terminata*; for details see [36].

A further insight in DQA that clashes with the stenographic narrative (a.k.a. syncategorematic interpretation) is Leibniz's insight that it is specifically in *nature* that bounded infinities are impossible. Here Leibniz observes that, though it may be impossible to find bounded infinities in nature (*rerum natura*), their usefulness to the geometer is independent of the metaphysician's task of elucidating their relation to natural phenomena:

> "Determining whether nature warrants quantities of this type is the business of the Metaphysician; for the Geometer it shall suffice to demonstrate what follows from their supposition."

("An autem hujusmodi quantitates ferat natura rerum Metaphysici est disquirere; Geometra sufficit, quid ex ipsis positis sequatur, demonstrare." [46, p. 98]).

To Leibniz, the impossibility of bounded infinities amounts to their absence in nature, not to any contradictory character as per the Arthur–Rabouin narrative. On varieties of *impossibility* in Leibniz, see Section 5.9.

4. An academic echo chamber

4.1. **Archibald.** Historian Tom Archibald happens to be the first-mentioned author of a 9-author cartel whose driving force was Rabouin (see Section 5.8), which published a 3-page broadside [1] against our article [32] on Leibniz in *The Mathematical Intelligencer* in 2022. The participants of the echo chamber are Archibald, Arthur, Ferraro, Gray, Jesseph, Lützen, Panza, Rabouin, and Schubring.

The title of the broadside is quite something: 'A Question of Fundamental Methodology: Reply to Mikhail Katz and His Coauthors'. Here one reads: "Leibniz's main argument is that it is not possible to treat infinitesimals as existing entities because that amounts to the introduction of an infinite number, which he takes to be a contradictory notion" [1, p. 361]. But such a reading of Leibniz aligns only with the Rabouin–Arthur narrative, according to which "infinitesimals, like infinite wholes, cannot be regarded as existing because their concepts entail contradictions" [53, Abstract].

Actually what Leibniz held to be contradictory are 'infinite wholes', which is only one of the possible meanings of the term *infinite number* in Leibniz. A different meaning was developed by Leibniz in his *De Quadratura Arithmetica* and elsewhere. This is the *infinitum terminatum*, a type of 'infinite number' that was further developed by late 19th century scholars like Levi-Civita, as pointed out by Laugwitz (see Section 2). For the Leibnizian distinction between *infinitum terminatum* and *infinitum interminatum*, see Section 3 and Section 5.8. Both the *infinita terminata* and infinitesimals are fictional non-contradictory notions. This has consistently been our position. Thus Archibald et al.'s claim about the contradictoriness of *infinite number* involves equivocation on the meaning of the term.

Archibald et al. go on to manufacture a series of misrepresentations of our work, as when they claim that "Nonstandard techniques of the kind invented by Robinson were presented [by Katz and coauthors] as a way to extend the realm of fictions related to infinite cardinalities by dealing with them in a consistent logical theory" [1, p. 362]. Archibald's claimed connection between fictional infinitesimals and infinite cardinalities is inaccurate. As we made abundantly clear in our work, infinite cardinalities (which would certainly have been rejected by Leibniz as contradicting the part-whole principle which he viewed as axiomatic) are not necessary for developing a theory of infinitesimals. In sum, the argument of [1] is based on the Rabouin–Arthur claim that infinitesimals were contradictory. Curiously, such a claim appears nowhere in the work of their coauthor Jesseph (quite the contrary); see Section 5.1.

4.2. **Argentina.** The year after Rabouin and Arthur published their syncategorematic take [53] on Leibniz in the *Archive for History of Exact Sciences* in 2020, an Argentinian team (Esquisabel and Raffo Quintana) published, in the same journal, a perceptive study [22] of Leibniz. Their study contains an explicit refutation of Rabouin and Arthur, though in passages hidden well in the middle of their text. Specifically, they reject the interpretation of Leibnizian infinitesimals as contradictory.

4.3. **Arthur.** Philosopher Richard Arthur is a veteran syncategorematist; see e.g., his "Leibniz's syncategorematic infinitesimals" [2]. As late as 2024, Arthur is still committing short-sighted errors of interpretation to prop up the stenographic narrative he inherited from Ishiguro's 1990 book [24]; see Section 5.9.

4.4. **Bos.** Historian Henk Bos is the author of a seminal 1974 article [16] on Leibniz. Bos also commented on the issue of interpreting Leibnizian calculus in Abraham Robinson's framework. On page 13 he wrote:

> A preliminary explanation of why the calculus could develop on the insecure foundation of the acceptance of infinitely small and infinitely large quantities is provided by the recently developed non-standard analysis, which shows that it is possible to remove the inconsistencies without removing the infinitesimals themselves. [16, p. 13]

This comment appears to endorse in principle the idea of using modern infinitesimal analysis to clarify some problems with the Leibnizian calculus. Meanwhile, Bos' Appendix 2 starts 68 pages later and tends to walk back the tentative endorsement of page 13. Since the article was published in the same year Robinson died, one wonders about the sequence of events that may have led to the addition of the appendix.

One thing is certain, however: Bos' seminal article contains not a trace of syncategoremania. Given the recognized seminal nature of Bos' study, this a huge problem for syncategorematists of Rabouin's ilk. The writings of Arthur and Rabouin contain only lame attempts to explain this away by means of far-fetched readings of Bos.

4.5. **Dauben.** Joseph Dauben is a historian of mathematics and the author of several well-received books [18], [17], [19], [20] and numerous articles. Dauben asks:

> Is it anachronistic to use nonstandard analysis or transfinite numbers to "rehabilitate" or explain the works of Leibniz, Euler, Cauchy, or Peirce, for example, as recent mathematicians, historians, and philosophers of mathematics have attempted? [21, p. 307]

He answers as follows:

> Robinson succeeded in showing the reasonableness of "redrawing" the early history of the calculus to reinstate past views that, cast in the light of nonstandard analysis, could be seen more clearly. In these cases at least, an anachronistic explanation nevertheless serves to clarify, not confound, what had confused earlier defenders

> of theories based on infinitesimals like the calculus. [21, p. 327]

While Dauben's appreciation of the clarifying role of nonstandard analysis as applied to the history of the calculus is certainly refreshing (especially when contrasted with broadsides and mockery coming from the cartel), it is important to note that there is nothing anachronistic about applying nonstandard analysis to clarify the *procedures* of the historical infinitesimalists, as opposed to *foundational* issues involved in how exactly such procedures are to be formalized. More precisely, applying a Robinsonian framework to this end is no more anachronistic than applying the more traditional Weierstrassian framework to this end. Both post-Weierstrassian classical analysis and infinitesimal analysis in the Robinsonian school are grounded in set-theoretic *foundational* developments which in themselves shed no light on the thinking of Leibniz, Euler, Cauchy and other infinitesimalists. But the *procedures* of modern infinitesimal analysis provide closer proxies for their procedures than the traditional post-Weierstrassian procedures. The distinction was elaborated in [10] and [5] using Benacerraf [12] as a starting point. See further in [6] on procedures.

4.6. **Eklund.** Samuel Henry Eklund is a student of Jeremy Heis. There is something of a mystery surrounding his 2020 doctoral thesis on Leibniz. The thesis can be found at `https://escholarship.org/uc/item/4732x76q` and it decisively breaks with the syncategorematic interpretation. This is not likely to have pleased everybody. There is no evidence Eklund continued in an academic career. Heis is keeping mum and not responding to emails. Was Eklund canceled? None of the familiar names dominating Leibniz scholarship (and none of the cartel of 9) appear as readers of his thesis or members of his committee.

4.7. **Ishiguro.** Philosopher Hidé Ishiguro wrote a book on Leibniz but she confused l'Hôpital and Nieuwentijt in her book (l'Hôpital was an ally; Nieuwentijt was an opponent). When M. Katz sent the paper [4] exposing this historical blunder to historian Mancosu, he responded: "I could not believe the mistake Ishiguro makes replacing Nieuventijt with L'Hôpital." A detailed rebuttal of Ishiguro's case for syncategorematics appears in [10].

5. From Jesseph to Schubring

5.1. **Jesseph.** Douglas Jesseph is one of the cartel of 9. In 1989, he saw Leibniz as having two ways of interpreting the calculus:

> Rather than postulate an ambiguity in Leibniz' writings, we can take him at his word: he is convinced that infinitesimal magnitudes are eliminable, and the reason for this conviction is the thoroughly commonsensical belief that the truth of his mathematical results should not depend too heavily upon the resolution of metaphysical problems. In the Leibnizian scheme, true mathematical principles will be found acceptable on any resolution of the metaphysical problems of the infinite. Thus, Leibniz' concern with matters of rigor leads him to propound a very strong thesis indeed, namely no matter how the symbols "dx" and "dy" are interpreted, the basic procedures of the calculus can be vindicated. (Jesseph [25, p. 243])

Jesseph concludes:

> Such vindication could take the form of a new science of infinity, or it could be carried out along classical lines, but in either case the new methods will be found completely secure. (Ibid.)

Accordingly, infinitesimals are not merely an abbreviated way of speaking (as per Ishiguro) and certainly not contradictory (as per Rabouin and Arthur [53]), but rather well-founded fictions within 'a new science of infinity'. In 1998, Jesseph explains the difference between well-founded and ill-founded fictions, and concludes:

> Infinitesimal magnitudes ... are nevertheless well-founded fictions precisely because the realm of mathematical objects is structured just as it would be if the infinitesimal existed. (Jesseph [26, pp. 35–36])

In these passages, Jesseph clearly distances himself from the syncategorematic interpretation according to which infinitesimals cannot constitute a 'new science'; nor could the realm of mathematical objects be structured "just as it would be if the infinitesimal existed." It is therefore surprising that he should have consented to have his name added to the author list of the broadside [1] endorsing the syncategorematic interpretation; see Section 4.1.

In 2008, Jesseph finds a source for Leibnizian infinitesimals in the notion of *conatus* of Hobbes:

> Hobbes was one among many whose writings stimulated the development of the Leibnizian approach to the calculus. However, there is one important difference between

> the Leibnizian and Hobbesian conceptions of *conatus* that is significant: Leibniz's language (at least in the *Theoria motus abstracti*) strongly implies that *conatus* be a literally infinitesimal quantity, while Hobbes regards it as having finite magnitude, but one so small as to be disregarded. In the end, however, Leibniz adopted a doctrine not far removed from Hobbes's. (Jesseph [27, p. 225])

Actually, Jesseph's argument based on Leibniz's 1671 text *Theoria motus abstracti* is unconvincing since the text dates from an early phase of Leibniz's development when his views were markedly different from his mature views that first emerged in 1676; see [57]. We mention it here to illustrate Jesseph's own position. Viewing Leibniz's 'literally infinitesimal quantity' as inspired by Hobbesian small negligible *conatus* leaves little room for claiming that the term does not *refer* (in Ishiguro's sense).

5.2. **Jesseph in 2015.** In 2015, Jesseph asserts that "a central task for Leibniz's fictionalism about infinitesimal magnitudes is to show that their introduction is essentially harmless in the sense that it *does not yield a contradiction*" [28, p. 196] (emphasis added). That's not what the broadside [1] said; see Section 4.1.

After outlining putative syncategorematic accounts (in terms of auxiliary curves and exhaustion) of Proposition 6 of Leibniz's text *De Quadratura Arithmetica* by Arthur, Knobloch and Levey, Jesseph objects in the following terms:

> The problem is that the construction of the auxiliary curves requires that we have a tangent construction that will apply to the original curve. This is readily available in the case of the circle, and tangents to conic sections and other well-behaved curves are also constructible with classical methods. However, one great strength of the infinitesimal calculus is that it enables algorithmic solutions to problems of tangency for a wide variety of curves; yet the procedure in the *Arithmetical Quadrature* could only be made fully general if we already had a solution to the general problem of tangent construction. (Jesseph [28, p. 199])

Accordingly, Leibniz's exhaustion argument ultimately relies on genuine infinitesimals, required for solving the problem of tangency in the general case. Jesseph's analysis reveals that he feels that syncategorematists don't have a convincing account of Leibniz's procedure beyond the simplest curves like conic sections. Jesseph continues:

> I suspect that he set aside the *Arithmetical Quadrature* without publishing it because he had turned his attention to more powerful methods that he would introduce in the 1680s in what he called "our new calculus of differences and sums, which involves the consideration of the infinite", and "extends beyond what the imagination can attain". (Jesseph [28, p. 200])

Accordingly, it was precisely 'the consideration of the infinite' (rather than a rejection thereof) that was behind the advantage of the 'new calculus' over whatever *De Quadratura Arithmetica* had to offer. With regard to Leibniz's alternative method relying on a Law of Continuity, Jesseph writes:

> it seems that the infinitesimal here is introduced as something like a Hilbertian "ideal element" that arises when we consider limit cases and seek what Leibniz termed "the universality which enables [the calculus] to include all cases, even that where the given lines disappear." He insisted that "it would be ridiculous not to accept [these ideal or limit cases] and so to deprive ourselves of one of [the calculus'] greatest uses" ... Leibniz frequently drew an analogy between such "ideal" elements as infinitesimals and the "imaginary" roots of negative numbers in algebra. The notion at work here is that, just as fictional quantity such as $\sqrt{-1}$ can be used to find the solution to a problem in trigonometry, we can also employ infinitesimal increments such as dx to arrive at solutions to geometric problems, ... (Jesseph [28, p. 202])

Here Jesseph compares Leibnizian infinitesimals and Hilbertian 'ideal elements'. Such a comparison was one of the main themes of Sherry and Katz [56] and would be unacceptable to syncategorematists. Jesseph's comparison of infinitesimals and imaginary roots is particularly significant. There is no known 'syncategorematic' account capable of justifying the use of $\sqrt{-1}$. The same goes for Leibnizian infinitesimals.

5.3. **Jesseph on incomparables.** In connection with the notion of incomparable infinitesimal, Jesseph quotes the following Leibnizian passage dating from 1689:

> "I have assumed in the demonstrations *incomparably small* quantities, for example the difference between two *common* quantities which is incomparable with the quantities themselves. Such matters as these, if I am not mistaken, can be set forth most lucidly in what follows. And then if someone does not want to employ *infinitely small* quantities, he can take them to be as small as he judges sufficient to be incomparable, so that they produce an error of no importance and even smaller than any given [error]. Just as the Earth is taken for a point, or the diameter of the Earth for a line infinitely small with respect to the heavens, so it can be demonstrated that if the sides of an angle have a base incomparably less than them, the comprehended angle will be incomparably less than a rectilinear angle, etc." (Leibniz as translated by Jesseph [27, p. 227, n. 23]; emphasis on 'common' added)

Jesseph claims the following:

> "one might interpret Leibniz's reply to Nieuwentijt [which cites the 1689 text] as defending the reality of infinitesimals, seeing the term 'incomparably small' as a kind of euphemism for 'infinitesimal.' But I think such an interpretation ultimately fails." [27, pp. 226–227]

Why would such an interpretation fail? According to Jesseph, it would fail on the grounds that "the doctrine is illustrated by the example of taking the Earth as a point with respect to the heavens, precisely as Hobbes had done" [27, p. 228], which to Jesseph indicates that such ratios are only small rather than infinitesimal.

In this instance, Jesseph's usually keen intepretive sense has failed him, because he overlooked the term *common* used by Leibniz in the 1689 passage Jesseph quoted:

> 'the difference between two *common* quantities which is incomparable with the quantities themselves'.

When speaking of genuine - of course, fictional - infinitesimals, Leibniz sometimes used the qualifier *incomparable* (here 'genuine' means that they violate the Archimedean property when compared to 1, while 'fictional' means that they don't correspond to anything in either the physical world or any putative Platonic realm). Meanwhile, when speaking of 'real-life' examples such as the Earth compared to the heavens, Leibniz used the term 'common incomparables'. These serve as motivating examples for genuine incomparable infinitesimals, rather than being an account of the infinitesimals themselves.

In sum, throughout a series of publications on Leibniz spanning a quarter of a century, Jesseph consistently proposed interpretations of Leibniz's fictional infinitesimals that are incompatible with the syncategorematic narrative. Jesseph generally refrained from endorsing the latter, but only until the broadside [1].

5.4. **Malet.** Antoni Malet was an Editor-in-Chief of the journal *Historia Mathematica* when one of our articles on infinitesimals was submitted there. He found it appropriate to forward to us a referee report whose conclusion is worth quoting in full (reproduced verbatim including spelling errors):

> I was about to call the manuscript a slapdash collection of bits and pieces randomly strung together. But that would be too generous. The authors have not even attempted to string their scatterbrain thoughts together. The have merely placed a bunch of random scraps adjacent to each other and submitted the resulting monstrosity as a research article.

Recall that this is a referee report for what is supposed to be a serious scholarly publication venue. The 'scatterbrain monstrosity' in question went on to be published in the *British Journal for the History of Mathematics* based on detailed and favorable referee reports. The precise reference is [33].

Another of Malet's editorial decisions was to desk-reject our response to an attack on our work by Arthur and Rabouin [3] in *Historia Mathematica*, on the grounds that the journal does not publish responses, even though Knobloch's response [38] to Blåsjö [13] duly appeared in *Historia Mathematica*. Our response eventually appeared as [34].

5.5. **Oaks.** Jeffrey Oaks is a mathematician-turned-historian. His case is particularly disappointing. Oaks and M. Katz had several discussions of the history of mathematics. While he repeatedly pointed out that he specialized in an earlier period more-or-less ending with Vieta, he seemed consistently sympathetic to our viewpoint on Fermat and later authors. Then something changed. Katz sent him a link to a short 1695 letter from Leibniz to l'Hôpital, which makes it clear that Leibniz viewed his ('incomparable') infinitesimals as violating the Archimedean property. Oaks replied: "I cannot do justice to this issue by reading one passage. Besides, I am absolutely swamped with work right now. But I have saved it for future reference." This reminds one of nothing more than the scene from Brecht's play on Galileo (probably apocryphal) where Galileo is trying to convince the advisers of the Grand Duke of Medici to observe Jupiter's moons through a telescope, and after a long deliberation they refuse to do so for weighty reasons of state. Soon enough, it turned out that Oaks had in the meantime been named as an Editor-in-Chief of the journal *Historia Mathematica*.

5.6. **Panza.** Marco Panza is the only member of the cartel of 9 who was willing to engage in a substantive email exchange concerning the controversy over Leibniz and related issues. After a few email exchanges, Panza sent the following comment (reproduced verbatim including spelling errors):

> What I contest is not at all this diversity of opinions, but the idea that the task of an historian is to attack different interpretations than his/her ones, to make his/her one finally triomphe. A critical discussion of different interpretations is useful but the final aim is advancing an interpretation, not destroying the others.

Katz responded by pointing out the following:

> I would like to understand further your distinction between 'attacking other interpretations' and 'critical discussion of other interpretations'. Let's consider two examples. 1. Rabouin [one of the cartel of 9] claims in the TMI 'Viewpoint' that our interpretation of Leibniz contains internal contradictions. Is that 'attacking' or 'critically discussing'? 2. In our articles [such as [5]], we have pointed out that Jeremy Gray's remarks [Gray is one of the cartel of 9] to the effect that 'Euler's foundations are dreadful' are unsupported, though consistent with received views on the history of mathematics in terms of teleology culminating in the triumph of what Boyer described as 'the great triumvirate' [Cantor, Dedekind, Weierstrass]. We also provided detailed arguments why we think Euler's foundations are by no means dreadful. Is this 'attacking' or 'critically discussing'?

That was the last we heard from Panza on this subject. Like Jesseph (see Section 5.1), Panza contradicted what he wrote in his own earlier scholarship concerning Leibniz when he joined the cartel. In 1992, Panza contrasts Newton's tradition with

> La deuxième tradition ... que j'ai appelée *infinitésimaliste* et qui remonte aux travaux de Leibniz et Johann I Bernoulli:

> le *calcul* est considéré comme un algorithme des différences infiniment petites qui se produisent dans une certaine quantité lorsqu'une différence de la même sorte se produit dans une quantité liée. (Panza [47, p. xix]; emphasis in the original)

Here Panza includes both Leibniz and Bernoulli in the same 'infinitesimal tradition'. Since it is generally acknowledged that for Johann Bernoulli, infinitesimals were mathematical entities (rather than non-'referring' stenography for exhaustion), Panza's grouping of Leibniz with Bernoulli in his description of the second, 'infinitesimalist' tradition puts Panza at odds with his coauthor Rabouin's stenographic narrative.

5.7. **Probst.** Historian Siegmund Probst is one of a handful of Leibniz scholars working on the Academy edition of the complete works of Leibniz; see `https://leibnizedition.de`. Probst writes:

> [T]he question is whether Leibniz based his metaphysical foundation of the calculus wholly on the concept of the syncategorematic infinite or whether he pursued also an alternative approach of accepting infinitesimals similar to what is done in modern nonstandard analysis. (Probst [48, p. 221])

Then he notes in a footnote: "Opposing views are stated in, e.g., Arthur (2013), *Leibniz's syncategorematic infinitesimals*; Sherry and Katz (2012), *Infinitesimals, imaginaries, ideals, and fictions*." The reference is respectively to Arthur [2] and Sherry and Katz [56]. Leibniz historian Probst evidently is willing to contemplate alternatives to the syncategorematic narrative.

5.8. **Rabouin.** David Rabouin published a 2017 article [51] containing an endorsement of the syncategorematic interpretation of Leibnizian infinitesimals. But we were able to locate a preliminary 2013 version [49] of the article which supports an opposing view. This enables us to pinpoint fairly accurately the moment when Arthur (see Section 4.3) "converted" Rabouin to the stenographic narrative. This pair of texts was analyzed in [34]. In fact he had already endorsed the stenographic narrative by 2015; see [50, pp. 361–362].

Why would Arthur and Rabouin insist (in both [1] and earlier and later work) on infinitesimals being contradictory?

To clarify this issue, we will use as a starting point the fact that, as many scholars agree, Cauchy did not distinguish between ordinary continuity and uniform continuity, and his definitions of continuity were sufficiently ambiguous to accommodate such a doubt. One could criticize such an insight on the grounds that it is ahistorical since the distinction (between two types of continuity) did not exist in Cauchy's time (or perhaps existed only toward the end of his life). Nonetheless many scholars accept such a diagnosis of Cauchy's stance on continuity.

What we see is that a modern concept C (such as the distinction between two types of continuity) can have important clarifying power even when applied to historical mathematicians who did not possess C.

Applying such an analysis to the Arthur–Rabouin situation, we observe that their problem is that they can't see the difference between infinite cardinals and ringinals. Here an infinite *ringinal* is exemplified by a natural number bigger than any naive counting number. A typical example of a ringinal is a nonstandard integer in $\mathbb{N}$.

In his *De Quadratura Arithmetica* and elsewhere, Leibniz distinguished between *infinitum terminatum* (literally, 'bounded infinity') and *infinitum interminatum* ('unbounded infinity'); see Section 3. The significance of the distinction was emphasized already in the 1980s and 1990s by Leibniz scholar Knobloch [37, p. 97], though his account of *bounded infinity* is flawed; see [36] and references cited there.

An unbounded infinity in modern terms would be a cardinal, whereas a bounded infinity would be a ringinal. Of course, neither notion (cardinal or ringinal) crystallized completely in Leibniz's time.

Now an unbounded infinity, also known as an *infinite whole*, is a contradictory concept according to Leibniz. Sometimes he refers to it as a *maximum*. The 'reciprocal' counterpart of a *maximum* is a *minimum* which is a point viewed as a constituent part of the continuum. A *minimum* is similarly an incoherent notion for Leibniz since Leibniz rejected the punctiform continuum (that a majority of mathematicians take for granted today).

Meanwhile, according to Leibniz a bounded infinity, say H, is not contradictory, and is in fact useful in geometry. An infinitesimal is the reciprocal of such an H.

Now if any notion of infinity, according to Arthur–Rabouin, must be an infinite whole (or in modern terminology an infinite cardinal), bounded infinities (in modern terminology, ringinals) must also be contradictory, as well as their reciprocals which are infinitesimals. Their lack of appreciation for the Leibnizian distinction seems to be what is behind their persistent claim that infinitesimals are contradictory.

Rabouin and Arthur succumb to some significant conflations. We point out three major errors in their argumentation:

(1) the conflation of existence in nature and existence in mathematics,
(2) the conflation of infinite wholes and bounded infinities (*infinita terminata*),
(3) the conflation of homogeneity and comparability.

For details see [34].

5.9. **Rabouin–Arthur on Froidmont and impossibility.** One of the most short-sighted blunders in Arthur–Rabouin is their treatment in [3] (2024) of Leibniz's notion of impossibility. They quote Leibniz as follows:

> "Therefore at some time the ratio of what is traversed to the unassignable whole must necessarily begin to be assignable, but such a point or instant in which that transition comes about is *impossible*. Therefore such a motion, and so such a line, must also be held to be *impossible*, and there is no infinite yet bounded straight line. Whence it also follows that infinitely small straight lines are *chimeras*, although nevertheless useful, like imaginary roots ..." (Leibniz as translated by Arthur and Rabouin in [3, p. 39]; emphasis added)

This is from a Leibnizian text referred to as "notes on Froidmont" dating from the mid-1690s. They proceed to claim the following inference:

> There is no ambiguity about the conclusion here: Leibniz considered such a reasoning as establishing the impossibility of infinite bounded lines, so that they should be ranked with infinitely small lines in the store of chimerical mathematical entities. But the most important fact is to note that from 1676 to the reflections on Froidmont, we see Leibniz considering infinitesimal lines as absurd, even when not taken as genuine magnitudes. As we have established, he attempted regularly to prove that they involve a contradiction. (Arthur and Rabouin [3, p. 39])

Arthur and Rabouin claim that Leibniz attempted to prove that infinite bounded lines "involve a contradiction" but an attentive reader will notice that the Leibnizian passage mentions no 'contradiction' at all. Rather, Leibniz uses the terms 'impossible' and 'chimeras'. But what is the difference between *impossibility* and *contradiction*?

To understand the distinction, we go back a decade to Leibniz's 1683 text *Elementa nova matheseos universalis* where Leibniz mentions that imaginary roots do not exist in *rerum natura*, and contrasts imaginary quantities *impossible by accident*, on the one hand, and *absolutely impossible* entities involving a contradiction such as $3 = 4$, on the other. Leibniz goes on to compare imaginary quantities (impossible by accident) to infinitely small quantities, and points out that people who are not sufficiently expert tend to confuse apparent impossibility with absurdity.

Absolute impossibility amounts to contradictoriness, but *accidental* impossibility only entails non-existence in *rerum natura*. Bounded infinities and infinitesimals are accidentally impossible. When Leibniz speaks in the notes on Froidmont about the impossibility of bounded infinities and infinitesimals, he has accidental impossibility in mind, rather than contradiction/absolute impossibility, since the context of his notes is an analysis of the physical realm. Arthur and Rabouin concede that "The general context is a part of the notes in which Leibniz goes back to the possibility of interpreting physical *conatus* as infinitely small lines traversed in a moment" [3, pp. 38–39]. Indeed, the passage speaks of *motion*, a physical notion.

Libert Froidmont (1587–1653) was a theologian interested primarily in the structure of the *physical* continuum, as is evident from the discussion in Beeley [11, p. 301]. Leibniz in his mature phase clearly distinguished between the physical realm, on the one hand, and mathematics, on the other. Beeley explains how Froidmont defends the idea of a non-atomistic continuum, drawing heavily on Aristotle. Froidmont offers physical arguments in support of his view. For example, he argues that motion, space, and time are inseparably linked, so if one (like motion) were discontinuous, the others would have to be as well. To sustain their continuity, he treats continuity as a fundamental principle of nature–though this introduces a degree of circular reasoning, since it assumes what it aims to prove. He also argues that the continuum cannot consist of points conceived as indivisible entities without parts, since there would have to be a small vacuum between them, but nature does not allow for a vacuum. Froidmont's line of reasoning shows that what Froidmont is referring to is the physical continuum.

Accordingly, the issue in Leibniz's notes is *physical* reality. Leibniz wonders whether bounded infinities and infinitesimals could be found there, the answer being negative. Mathematical infinitesimals, while accidentally impossible, involve no contradiction. In 2026 Rabouin persists in conflating distinct notions of infinity, and distinct types of impossibility, when he describes the Leibnizian position as follows: "there is no 'infinite number, nor infinite line or other infinite quantity' (since all of these objects are defined by Leibniz in terms of 'genuine wholes' and formed by addition of parts)" [52, p. 2]. For details on accidental and absolute impossibility in Leibniz see [31, Section 5.3].

In 2026, Rabouin [52, p. 2] quotes Leibniz's *New Essays* of 1704 to the effect that

> (*) these infinite wholes, and their opposites the infinitesimals, have no place except in the calculations of geometers, just like the imaginary roots in algebra.

This passage seems to suggest that infinitesimals are reciprocals ('opposites') of infinite wholes, i.e., of *infinita interminata*, rather than of *infinita terminata*. On the other hand, there is something odd about the passage. The usefulness of infinitesimals and imaginaries is a familiar theme in Leibniz, but the passage seems to include infinite wholes in the same category of usefulness as infinitesimals and imaginaries. This is contrary to many of Leibniz's *mathematical* texts emphasizing the fact that infinite wholes are *not* useful (because contradictory). Regardless of whether one interprets Leibnizian infinitesimals as genuine or as stenographic, there is a problem with this passage that is not addressed by Rabouin: infinite wholes and infinitesimals are lumped together as useful, whereas elsewhere Leibniz clearly distinguishes between them:

> (**) infinite wholes are useless, infinitesimals are useful.

How are we, then, to understand the passage (*)? *New Essays* is not a mathematical text and the passage dealing with infinite wholes and infinitesimals is merely a brief digression contained in a paragraph dealing with theological matters and the status of infinite wholes in that particular domain of thought. Since, on mathematical issues, Leibniz's mathematical texts should clearly take precedence over a treatise dealing primarily with non-mathematical matters, the presumption (**) remains in force, and we can account for the passage (*) by taking the Leibnizian mention of 'infinite wholes' in (*) as being a slightly imprecise reference to infinite *numbers* (rather than wholes) which in the context of the passage necessarily refer to *infinita terminata* (and its opposites the infinitesimals). Leibniz used the term 'infinite whole' only because it was used earlier in the same paragraph in a theological discussion.

To summarize, the meaning of (*) is that infinitesimals and their reciprocals, as well as imaginary roots, do not occur in nature, but are useful in geometry and Algebra (and therefore cannot be contradictory). Leibniz had also written this in many other texts. On the other hand, he repeatedly emphasized that he considers infinite wholes to be contradictory (and therefore mathematically useless). Therefore, these cannot be the reciprocals of infinitesimals. We can thus only understand (*) to mean that Leibniz is referring here to what he has called *infinite numbers* or *infinita terminata* elsewhere.

Rabouin's 2026 text [52] contains numerous additional conflations, equivocations, inaccuracies, misrepresentations and misunderstandings. Both his abstract and his introduction offer effusive praise to Eberhard Knobloch as contributing to understanding "Leibniz's conception of mathematical infinity" [52, p. 1]. What Rabouin fails to mention is Knobloch's emphasis on the distinction between the Leibnizian *infinita terminata* and *infinita interminata*, as elaborated in several articles from the 1980s and 1990s; see Sections 3 and 5.8. Syncategorematists of Rabouin's ilk have no convincing way of accounting for this Leibnizian dichotomy.

5.10. **Schubring.** Gert Schubring is the last-mentioned member of the cartel of 9 [1], and the only one to have actually engaged in an exchange of articles. Our [15] included a section containing a critique of an aspect of Schubring's book [54]. Schubring responded in [55]. We responded in [14] with a more detailed critique of his book. Schubring's book contains scathing attacks against the remarkable scholarship [39], [40] of historian Detlef Laugwitz (see Section 2) published in leading history of mathematics and science journals, as well as mockery of nonstandard analysis as an interpretive tool. Thus, Schubring writes:

> Detlef Laugwitz ... has been analyzing Cauchy since the 1980s with the goal of ruling out every attribution of error to his work. In a discussion with Pierre Dugac in 1986, he formulated his maxim as follows: Cauchy was such a great mathematician. Everything else he did was correct. Therefore, when faced with a choice between two possible interpretations of his concepts, one has to go for the one that makes his proof correct. In 1984, Giusti published a decisive article on Cauchy's 'errors' ... This spurred Laugwitz to even more detailed attempts to banish the error and confirm that Cauchy had used hyper-real numbers. On this basis, he claims, the errors vanish and the theorems become correct, or, rather, they always were correct (see Laugwitz 1990, 21). (Schubring [54, p. 432])

This sarcastic slapdash is a caricature of Laugwitz's position and in particular of page 21 in his 1990 article [41] cited by Schubring, as we demonstrated in our critique of Schubring's approach; see [14].

Contrary to Schubring's claim, Laugwitz never asserted that Cauchy had used the hyperreals. Furthermore, Laugwitz never asserted that "errors vanish". Rather, he noted that in 1853, Cauchy strengthened the hypothesis of his erroneous 1821 theorem.

No reply to our critique [14] followed (apart from Schubring's unrefereed MathSciNet diatribe `https://mathscinet.ams.org/mathscinet/article?mr=4196072` against our article [7], accusing us of leading a 'crusade') until the broadside [1]. Our [8] and [34] contain a detailed rebuttal of all the claims made in the broadside.

6. Conclusion

Our research challenges the received narrative and aims to re-evaluate the history of infinitesimals. The resistance we have encountered exemplifies broader issues of academic inertia and protection of established views. While the institutional resistance to paradigm shifts exemplified by the reaction of the old guard of the cartel of 9 and their acolytes at *Historia Mathematica* has evidently been disappointing, our work has been recognized by the editors and referees of many professional history, mathematics, and philosophy journals, and by a number of younger historians who have shown interest in our analysis. A re-evaluation of the received assumptions about the history of mathematics is under way.

References

[1] Archibald, Tom; Arthur, Richard T. W.; Ferraro, Giovanni; Gray, Jeremy; Jesseph, Douglas; Lützen, Jesper; Panza, Marco; Rabouin, David; Schubring, Gert. A Question of Fundamental Methodology: Reply to Mikhail Katz and His Coauthors. *The Mathematical Intelligencer* **44** (2022), no. 4, 360–363.

[2] Arthur, Richard. Leibniz's syncategorematic infinitesimals. *Archive for History of Exact Sciences* **67** (2013), no. 5, 553–593.

[3] Arthur, Richard; Rabouin, David. On the unviability of interpreting Leibniz's infinitesimals through non-standard analysis. *Historia Mathematica* **66** (2024), 26–42.

[4] Bair, J.; Błaszczyk, P.; Ely, R.; Heinig, P.; Katz, M. Leibniz's well-founded fictions and their interpretations. *Mat. Stud.* **49** (2018), no. 2, 186–224.

[5] Bair, J.; Błaszczyk, P.; Ely, R.; Henry, V.; Kanovei, V.; Katz, K.; Katz, M.; Kutateladze, S.; McGaffey, T.; Reeder, P.; Schaps, D.; Sherry, D.; Shnider, S. Interpreting the infinitesimal mathematics of Leibniz and Euler. *Journal for General Philosophy of Science* **48** (2017), no. 2, 195–238.

[6] Bair, J.; Błaszczyk, P.; Ely, R.; Katz, M.; Kuhlemann, K. Procedures of Leibnizian infinitesimal calculus: An account in three modern frameworks. *British Journal for the History of Mathematics* **36** (2021), no. 3, 170–209.

[7] Bair, J.; Błaszczyk, P.; Heinig, P.; Kanovei, V.; Katz, M. Cauchy's work on integral geometry, centers of curvature, and other applications of infinitesimals. *Real Analysis Exchange* **45** (2020), no. 1, 127–150.

[8] Bair, J.; Borovik, A.; Kanovei, V.; Katz, M.; Kutateladze, S.; Sanders, S.; Sherry, D.; Ugaglia, M. Historical infinitesimalists and modern historiography of infinitesimals. *Antiquitates Mathematicae* **16** (2022), 189–257.
[9] Bair, J.; Katz, M.; Sherry, D. Fermat's dilemma: Why did he keep mum on infinitesimals? and the European theological context. *Foundations of Science* **23** (2018), no. 3, 559–595.
[10] Bascelli, T.; Błaszczyk, P.; Kanovei, V.; Katz, K.; Katz, M.; Schaps, D.; Sherry, D. Leibniz versus Ishiguro: Closing a Quarter Century of Syncategoremania. *HOPOS: The Journal of the International Society for the History of Philosophy of Science* **6** (2016), no. 1, 117–147.
[11] Beeley, Philip. *Kontinuität und Mechanismus: Zur Philosophie des jungen Leibniz in ihren ideengeschichtlichen Kontext.* Studia Leibnitiana Supplementa 30. Verlag, Stuttgart, 1996.
[12] Benacerraf, Paul. What numbers could not be. *Philos. Rev.* **74** (1965), 47–73.
[13] Blåsjö, Viktor. On what has been called Leibniz's rigorous foundation of infinitesimal geometry by means of Riemannian sums. *Historia Mathematica* **44** (2017), no. 2, 134–149.
[14] Błaszczyk, P.; Kanovei, V.; Katz, M.; Sherry, D. Controversies in the foundations of analysis: Comments on Schubring's Conflicts. *Foundations of Science* **22** (2017), no. 1, 125–140.
[15] Błaszczyk, P.; Katz, M.; Sherry, D. Ten misconceptions from the history of analysis and their debunking. *Foundations of Science* **18** (2013), no. 1, 43–74.
[16] Bos, H. J. M. Differentials, higher-order differentials and the derivative in the Leibnizian calculus. *Archive for History Exact Sciences* **14** (1974), 1–90.
[17] Bos, H. J. M.; Bunn, R.; Dauben, Joseph W.; Grattan-Guinness, I.; Hawkins, Thomas W.; Pedersen, Kirsti Møller. *From the calculus to set theory, 1630–1910. An introductory history.* Edited by I. Grattan-Guinness. Gerald Duckworth, London, 1980. iii+306 pp.
[18] Dauben, Joseph Warren. *Georg Cantor. His mathematics and philosophy of the infinite.* Harvard University Press, Cambridge, Mass.-London, 1979. xii+404 pp.
[19] Dauben, Joseph W. *The history of mathematics from antiquity to the present. A selective bibliography.* Bibliographies of the History of Science and Technology, 6. Garland Reference Library of the Humanities, 313. Garland Publishing, New York, 1985. xxxix+467 pp.
[20] Dauben, Joseph Warren. *Abraham Robinson. The creation of nonstandard analysis. A personal and mathematical odyssey.* With a foreword by Benoit B. Mandelbrot. Princeton University Press, Princeton, NJ, 1995. xx+560 pp.
[21] Dauben, Joseph W. Anachronism and incommensurability: words, concepts, contexts, and intentions. In *Anachronisms in the History of Mathematics: Essays on the Historical Interpretation of Mathematical Texts*, 307–357, Cambridge Univ. Press, Cambridge, 2021.
[22] Esquisabel, Oscar; Raffo Quintana, Federico. Fiction, possibility and impossibility: Three kinds of mathematical fictions in Leibniz's work. *Archive for History of Exact Sciences* **75** (2021), no. 6, 613–647.
[23] Gerhardt, C. (ed.) *Leibnizens mathematische Schriften.* A. Asher, Berlin and Halle, 1850–63.

[24] Ishiguro, Hidé. *Leibniz's philosophy of logic and language.* Second edition. Cambridge University Press, Cambridge, 1990.

[25] Jesseph, Douglas M. Philosophical theory and mathematical practice in the seventeenth century. *Stud. Hist. Philos. Sci.* **20** (1989), no. 2, 215–244.

[26] Jesseph, Douglas M. Leibniz on the foundations of the calculus: the question of the reality of infinitesimal magnitudes. Leibniz and the sciences. *Perspect. Sci.* **6** (1998), no. 1-2, 6–40.

[27] Jesseph, Douglas M. Truth in fiction: Origins and consequences of Leibniz's doctrine of infinitesimal magnitudes. In *Infinitesimal Differences. Controversies between Leibniz and his Contemporaries*, pp. 215–233. Edited by Ursula Goldenbaum and Douglas Jesseph. Walter de Gruyter, Berlin–New York, 2008.

[28] Jesseph, Douglas M. Leibniz on the elimination of infinitesimals. In *G.W. Leibniz, interrelations between mathematics and philosophy,* pp. 189–205, Archimedes, 41, Springer, Dordrecht, 2015.

[29] Kanovei, V.; Katz, M.; Kudryk, T.; Kuhlemann, K. A philosophical history of infinitesimals. *Revista Latinoamericana de Filosofía* **52** (2026), no. 1, 137–169. https://doi.org/10.36446/rlf478, https://arxiv.org/abs/2605.13102

[30] Katz, M.; Kuhlemann, K.; Sherry, D. A Leibniz/NSA comparison. *London Mathematical Society Newsletter* (2024), no. 512, 33–37. https://www.lms.ac.uk/sites/default/files/inline-files/NLMS_512%20Online_0.pdf http://arxiv.org/abs/2409.17154

[31] Katz, M.; Kuhlemann, K.; Sherry, D.; Ugaglia, M. Leibniz on bodies and infinities: rerum natura and mathematical fictions. *Review of Symbolic Logic* **17** (2024), no. 1, 36–66.

[32] Katz, M.; Kuhlemann, K.; Sherry, D.; Ugaglia, M.; van Atten, M. Two-track depictions of Leibniz's fictions. *The Mathematical Intelligencer* **44** (2022), no. 3, 261–266.

[33] Katz, M. Episodes from the history of infinitesimals. *British Journal for the History of Mathematics* **40** (2025), no. 2, 123–135.

[34] Katz, Mikhail; Kuhlemann, Karl. Leibniz's contested infinitesimals: Further depictions. *Ganita Bharati* **45** (2023), no. 1, 77–112. (published in 2025)

[35] Katz, M.; Schaps, D.; Shnider, S. Almost equal: The method of adequality from Diophantus to Fermat and beyond. *Perspectives on Science* **21** (2013), no. 3, 283–324.

[36] Katz, M.; Sherry, D.; Ugaglia, M. When does a hyperbola meet its asymptote? Bounded infinities, fictions, and contradictions in Leibniz. *Revista Latinoamericana de Filosofía* **49** (2023), no. 2, 241–258.

[37] Knobloch, Eberhard. Galileo and Leibniz: different approaches to infinity. *Archive for History of Exact Sciences* **54** (1999), no. 2, 87–99.

[38] Knobloch, Eberhard. Letter to the editors of the journal Historia Mathematica. Remarks on the paper by Blåsjö. *Historia Mathematica* **44** (2017), no. 3, 280–282.

[39] Laugwitz, Detlef. Infinitely small quantities in Cauchy's textbooks. *Historia Mathematica* **14** (1987), no. 3, 258–274.

[40] Laugwitz, Detlef. Definite values of infinite sums: aspects of the foundations of infinitesimal analysis around 1820. *Archive for History of Exact Sciences* **39** (1989), no. 3, 195–245.

[41] Laugwitz, Detlef. Das mathematisch Unendliche bei Cauchy und bei Euler. ed. Gert König, *Konzepte des mathematisch Unendlichen im 19. Jahrhundert* (Göttingen, Vandenhoeck u. Ruprecht, 1990), 9–33.

[42] Laugwitz, D. Debates about infinity in mathematics around 1890: the Cantor–Veronese controversy, its origins and its outcome. *NTM (N.S.)* **10** (2002), no. 2, 102–126.

[43] Leibniz, G.W. *Letter to l'Hospital*, 14/24 june 1695, in Gerhardt [23], vol. I, pp. 287–289.

[44] Leibniz, G.W. Responsio ad nonnullas difficultates a Dn. Bernardo Niewentiit circa methodum differentialem seu infinitesimalem motas. *Act. Erudit. Lips.* (1695). In Gerhardt [23], vol. V, pp. 320–328. A French translation by Parmentier is in [45, pp. 316–334].

[45] Leibniz, G.W. *La naissance du calcul différentiel. 26 articles des* Acta Eruditorum. Translated from the Latin and with an introduction and notes by Marc Parmentier. With a preface by Michel Serres. Mathesis. Paris: Librairie Philosophique J. Vrin, 1989. See `https://books.google.co.il/books?id=lfEy-OzaWkQC`

[46] Leibniz, G.W. *Quadrature arithmétique du cercle, de l'ellipse et de l'hyperbole*. Marc Parmentier (Trans. and Ed.) /Latin text by Eberhard Knobloch (Ed.), Paris: J. Vrin, 2004. See `https://books.google.co.il/books?id=fNTUULXHmQOC`

[47] Panza, Marco. La forma della quantità. Analisi algebrica e analisi superiore: il problema dell'unità della matematica nel secolo dell'illuminismo. Tome I, II. *Cahiers d'Histoire et de Philosophie des Sciences. Nouvelle Série*, 38, 39. Société Française d'Histoire des Sciences et des Techniques, Paris, 1992.

[48] Probst, Siegmund. "The Calculus." In M. R. Antognazza (Ed.), *The Oxford Handbook of Leibniz* (pp. 165–182). Oxford University Press, 2018.

[49] Rabouin, David. On mathematical style (2013), 32 pages. (First version of a paper to appear in a shorter form, under the title "Styles in mathematical practice", in K. Chemla & E. Fox-Keller, *Cultures without Culturalism in the Making of Scientific Knowledge*, Duke University Press, 2017.) Accessible at `https://www.academia.edu/download/40811452/Rabouin`

[50] Rabouin, David. Leibniz's rigorous foundations of the method of indivisibles Or how to reason with impossible notions. In V. Jullien (ed.), *Seventeenth-Century Indivisibles Revisited*, Science Networks, Historical Studies 49, Springer, Switzerland, 2015.

[51] Rabouin, David. Styles in mathematical practice. In *Cultures without Culturalism*, pp. 196–223, Duke University Press, Durham, NC, 2017.

[52] Rabouin, David. A note on mathematical infinity in Leibniz. *Historia Mathematica* **74** (2026), Paper No. 103191, 10 pp.

[53] Rabouin, David; Arthur, Richard. Leibniz's syncategorematic infinitesimals II: their existence, their use and their role in the justification of the differential calculus. *Archive for History of Exact Sciences* **74** (2020), 401–443.

[54] Schubring, Gert. *Conflicts between generalization, rigor, and intuition. Number concepts underlying the development of analysis in 17–19th Century France and Germany.* Sources and Studies in the History of Mathematics and Physical Sciences. Springer-Verlag, New York, 2005.

[55] Schubring, Gert. Comments on a paper on alleged misconceptions regarding the history of analysis: who has misconceptions? *Foundations of Science* **21** (2016), no. 3, 527–532.

[56] Sherry, David; Katz, Mikhail. Infinitesimals, imaginaries, ideals, and fictions. *Studia Leibnitiana* **44** (2012), no. 2, 166–192. (Article was published in 2014 even though the journal issue lists the year as 2012)

[57] Ugaglia, Monica; Katz, Mikhail. Evolution of Leibniz's thought in the matter of fictions and infinitesimals. In: Sriraman, B. (ed.) *Handbook of the History and Philosophy of Mathematical Practice*, pp. 341–384, Springer, Cham, 2024.