Explorations in Harmonic Analysis and other realms

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Abstracts

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Ordered structure for Cauchy–de Branges spaces

I plan to discuss how the classical de Branges theory of Hilbert spaces of entire function can be extended to a more general setting of Cauchy–de Branges spaces. In particular, new versions of de Branges Ordering Theorem for nearly invariant subspaces in Cauchy–de Branges spaces will be reviewed. This is joint work with A. Baranov and Yu. Belov.

Anton Baranov

St. Petersburg State University

Differentiation invariant subspaces in the space of infinitely differentiable functions

In the space of all infinitely differentiable functions on an interval (a, b) consider a differentiation invariant subspace and assume that the restriction of differentiation onto this subspace has discrete spectrum. Is it true that in this case the subspace is generated by the exponential monomials it contains? In general, the answer is negative, since the subspace may have the so-called "residual" part (all functions vanishing on some subinterval). In 2007 A. Aleman and B. Korenblum posed the spectral synthesis problem: is any invariant subspace generated by its residual part and the corresponding exponentials? We give a complete description of subspaces which admit spectral synthesis in terms of their spectra. The talk is based on joint works with A. Aleman and Yu. Belov.

Yurii Belov
St. Petersburg State University

The Newman-Shapiro problem

In 1966 D. Newman and H. Shapiro posed the following problem. Let G be a function from Fock space $\mathcal{F}$ and such that $e^{wz}G(z) \in \mathcal{F}$ for any $w \in \mathbb{C}$. Is it true that

$$\operatorname{span}\{e^{wz}G(z)\} = \{FG : FG \in \mathcal{F}\}?$$

Recently the author (joint with A. Borichev) has constructed a counterexample to this conjecture. On the other hand, we are able to show that if G satisfies some regularity conditions, then conjecture holds. These results are closely connected to some spectral synthesis problems in Fock space.

Andriy Bondarenko
Norwegian University of Science and Technology, Trondheim

Extreme values of the Riemann zeta function

We will discuss how large $|\zeta(s)|$ can be on the critical line. Main results and notable open problems will be outlined.

Alexander Borichev
Université d'Aix-Marseille

Weak embedding property, inner functions and entropy

We study the class of inner functions F characterized by the property that $|F(z)|$ is bounded from below by a function of the distance of z to the zero set of F. We establish several results on the factors of such functions in terms of the porosity of the boundary spectrum.

Efim Gluskin
Tel-Aviv University

A remark on the Ekeland-Hofer-Zehnder capacity

Recently Y. Ostrover and the speaker introduced a linearized version of the symplectic capacity. They show that it is an upper bound for all symplectic capacities, and that, on another hand, that on the class of centrally symmetric convex bodies it is bounded by four times the Ekeland-Hofer-Zehnder capacity. In the talk it will be shown how some elementary harmonic analysis leads to an example of a convex body with a logarithmic gap between these two quantities. No knowledge on the symplectic geometry is assumed.

Håkan Hedenmalm
Royal Institute of Technology, Stockholm

Planar orthogonal polynomials and boundary universality in the random normal matrix model

We obtain a new asymptotic expansion of the orthogonal polynomials in the context of exponentially varying weights. This goes beyond the classical works of Carleman and Suetin, where the domain was fixed and the weight as well ("hard-edge"). Here, the domain is obtained implicitly from the problem using an energy approach, or alternatively from an obstacle problem. As a consequence, we obtain the boundary universality law for soft edges, given by the error function. This reports on joint work with A. Wennman.

Alex Iosevich
University of Rochester

Erdos/Falconer problems and applications to classical analysis

We are going to describe some recent developments pertaining to the Erdos and Falconer distance problems in the discrete and continuous settings, respectively. We shall then describe some applications of the underlying techniques to problems in classical analysis, such as the existence and non-existence of exponential and Gabor bases.

Boris Kashin
Steklov Mathematical Institute, Moscow

On the decomposition of a given matrix into two submatrices with extremely small $(2,1)$ norm

Sergei Kislyakov
St. Petersburg Department of Steklov Mathematical Institute

Interpolation for intersections of certain Hardy-type spaces

This is a part of a joint work by the author and K. Zlotnikov.

Let (X_0, X_1) be a compatible couple of Banach spaces, and let Y_0, Y_1 be closed subspaces of X_0 and X_1. The couple (Y_0, Y_1) is said to be K-closed in (X_0, X_1) if, whenever $Y_0 + Y_1 \ni x = x_0 + x_1$ with $x_i \in X_i$, $i = 0, 1$, we also have $x = y_0 + y_1$ with $y_i \in Y_i$ and $\|y_i\| \leqslant C\|x_i\|$, $i = 1, 2$. We recall that K-closedness does occur in the scale of the Hardy spaces on the unit circle (viewed as subspaces of the corresponding Lebesgue spaces), but we are interested in the following two more complicated results (in them we assume that $1 < p < \infty$).

1) The couple $(H^p(\mathbb{T}^2), H^\infty(\mathbb{T}^2))$ is K-closed in the couple $(L^p(\mathbb{T}^2), L^\infty(\mathbb{T}^2)$ (Kislyakov and Xu, 1996).

2) For an inner function θ on the unit circle, the couple $(H^p \cap \theta\overline{H^p}, H^\infty \cap \theta\overline{H^\infty})$ is K-closed in $(L^p(\mathbb{T}), L^\infty(\mathbb{T}))$ (Kislyakov and Zlotnikov, 2018)

Surprisingly, the proofs of these facts are quite similar, signalizing that they may be particular cases of some general statement. Such a statement exists indeed and looks roughly like this. Again, here $1 < p, \infty$.

Theorem. Let (X, μ) be a space with a finite measure μ, let A and B be w^*-closed subalgebras of $L^\infty(\mu)$, and let C and D be closed subspaces of $L^p(\mu)$ that are moduli over A and B, respectively. **Under certain additional assumptions**, the couple $(C \cap D, C \cap D \cap L^\infty(\mu))$ is K-closed in $(L^p(\mu), L^\infty(\mu))$.

The **additional assumptions** say, in particular, that some analogs of the harmonic conjugation operator relative to the algebras A and B have the usual properties, as, for instance, is in the case of w^*-Dirichlet algebras. However, the condition for A and B to be w^*-Dirichlet is too restrictive (in particular, we do not insist that a multiple of μ represent some multiplicative linear functional on either A or B). Also, note that the proofs of statements 1) and 2) known previously relied upon the fact that, in those settings, the corresponding harmonic conjugations (or Riesz projections) were classical singular integral operators. In particular, the two proofs started with employing Calderón–Zygmund decomposition, which is not available in the generality adopted in the theorem. Also, some assumptions on the mutual position of the annihilators of C and D are required (in the context of statements 1 and 2, these assumptions are satisfied trivially).

Bo'az Klartag

Weizmann Institute of Science

Convex geometry and waist inequalities

We will discuss connections between Gromov's work on isoperimetry of waists and Milman's work on the M-ellipsoid of a convex body. It is proven that any convex body K in an n-dimensional Euclidean space has a linear image K_1 of volume one satisfying the following waist inequality: Any continuous map f from K_1 to $\mathbb{R}^d$ has a fiber $f^{-1}(t)$ whose $(n-d)$-dimensional volume is at least c^{n-d}, where $c > 0$ is a universal constant. Already in the case where f is linear, this constitutes a slight improvement over known results. In the specific case where $K = [0,1]^n$, one may take $K_1 = K$ and $c = 1$, confirming a conjecture by Guth. We furthermore exhibit relations between waist inequalities and various geometric characteristics of the convex body K.

Mihalis Kolountzakis

University of Crete

Tiling by translation and bases of exponentials

Over the past several decades mathematicians have been trying to understand which domains admit an orthogonal (or, sometimes, not) basis of exponentials of the form $e_\lambda(x) = e^{2\pi i \lambda \cdot x}$, for some set of frequencies Λ. (This sometimes goes by the name non-harmonic Fourier Analysis.) It is well known that we can do so for the cube, for instance, but can we find such a basis for the ball? The answer is no, if we demand orthogonality, but what if we just ask for an unconditional (Riesz) basis? This is still unknown.

It was clear from the beginning that this question has a lot to do with tiling by translation (i.e., with filling up space with no overlaps by translating around an object).

Fuglede originally conjectured that an orthogonal exponential basis exists if and only if the domain can tile space by translation. This has been disproved in its full generality but when one adds side conditions, such as, for instance, a lattice set of frequencies, or the space being a group of a specific type, or the domain being convex, or many other natural conditions, the answer is often unknown, and sometimes known to be positive or known to be negative. In short, this is a wide open area of research, branching out by varying the side conditions on the domain or the group in which the domain lives, or by relaxing the orthogonality condition or even allowing time-frequency translates of a given function to serve as basis elements (Gabor, or Weyl-Heisenberg, bases).

When working with both exponential bases and tiling problems the crucial object of study turns out to be the zero set of the Fourier Transform of the indicator function of the domain we care about. In particular we want to know how large structured sets this zero set contains, for instance how large difference sets it contains or what kind of tempered distributions it can support.

After a brief historical introduction to these subjects and their relations we will go over some of the recent results in these fields.

Sergei V. Konyagin

Steklov Mathematical Institute, Moscow

On subsequences of almost everywhere convergence of partial trigonometric Fourier sums

Let $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$, $L(\mathbb{T})$ be the set of all integrable functions $f : \mathbb{T} \to \mathbb{C}$. We associate with a function $f \in L(\mathbb{T})$ its trigonometric Fourier series

$$f \sim \sum_{k=-\infty}^{\infty} \widehat{f}(k)e^{ikx}, \quad \widehat{f}(k) = \frac{1}{2\pi}\int_{\mathbb{T}} f(x)e^{-ikx}dx.$$

For $n \in \mathbb{N}$ define the n-th partial sum of f as

$$S_n(f;x) = \sum_{k=-n}^{n} \widehat{f}(k)e^{ikx}.$$

Let $\exp^*(0) = 1$ and $\exp^*(r) = \exp(\exp^*(r-1))$ for a positive integer r. Recall that a sequence of positive integers $\{n_j\}_{j\in\mathbb{N}}$ is lacunary if there exists a number $\rho > 1$ such that $n_{j+1}/n_j \geqslant \rho$ for any $j \in \mathbb{N}$.

Theorem. There exists a real function $f \in L(\mathbb{T})$ such that for any almost everywhere convergent subsequence of partial sums $\{S_{n_j}(f)\}$ with a lacunary sequence $\{n_j\}$ we have

$$n_j \geqslant \exp^*\left((\log\log j)^{1-o(1)}\right) \quad (j \to \infty).$$

Thomas W. Körner
University of Cambridge

Rearranged series

The first result of Olevskii that I met was a beautiful argument on the rearrangement of orthogonal series. I will give his proof and then discuss some later developments.

Vladimir Lebedev
National Research University Higher School of Economics, Moscow

Self-mappings of the circle that preserve the Beurling algebra $A_*(\mathbb{T})$

Consider the Wiener algebra $A(\mathbb{T})$ of all continuous functions f on the circle $\mathbb{T}$ whose Fourier series converge absolutely. The norm on $A(\mathbb{T})$ is naturally defined by

$$\|f\|_{A(\mathbb{T})} = \sum_{k\in\mathbb{Z}} |\widehat{f}(k)|.$$

In 1953 Beurling and Helson proved their famous theorem on changes of variable that preserve the absolute convergence. Namely they showed that if φ is a self-mapping of $\mathbb{T}$ with the property that for every $f \in A(\mathbb{T})$ the superposition $f \circ \varphi$ is also in $A(\mathbb{T})$, then φ is linear, i.e., $\varphi(t) = \nu t + \varphi(0)$ (with $\nu \in \mathbb{Z}$).

An interesting subclass of $A(\mathbb{T})$ is the class $A_*(\mathbb{T})$ of functions f whose sequence of the Fourier coefficients has a monotone majorant in l^1. Setting

$$\|f\|_{A_*(\mathbb{T})} = \sum_{n\geqslant 0} \sup_{|k|\geqslant n} |\widehat{f}(k)|$$

we obtain an algebra as well, often referred to as the Beurling algebra. Do there exist nonlinear self-mappings φ of $\mathbb{T}$ that preserve $A_*(\mathbb{T})$? We present some partial results on the subject. In particular, it turned out that nonlinear C^1 mappings do not preserve $A_*(\mathbb{T})$. One of the ingredients of the proof is a recent result by the author on sets with distinct sums of pairs and arithmetic progressions.

Andrei Lerner
Bar-Ilan University

On the weak Muckenhoupt-Wheeden conjecture

We construct an example showing the sharpness of certain weighted weak type $(1,1)$ bounds for the Hilbert transform. This is joint work with Fedor Nazarov and Sheldy Ombrosi.

Alexander Logunov
Institute for Advanced Study, Princeton

Landis conjecture

Landis conjectured that if u is a solution to $\Delta u + Vu = 0$ in the whole plane $\mathbb{R}^2$, where V be a bounded measurable function in $\mathbb{R}^2$, then $|u|$ cannot decay faster than exponentially near infinity. Namely, if

$$|u(x)| \leqslant e^{-|x|^{1+\varepsilon}}$$

for some $\varepsilon > 0$, then $u \equiv 0$. It was shown by Meshkov that there is a counterexample to the conjecture with $|u(x)| \leqslant e^{-|x|^{4/3}}$ and bounded complex-valued V. Meshkov also showed that $|u|$ cannot decay near infinity as fast $e^{-|x|^{4/3+\varepsilon}}$ without being identically zero.

When V is real-valued Landis conjecture is true and we will discuss this recent result. Based on a joint work in progress with N. Nadirashvili, F. Nazarov, Eu. Malinnikova.

Eugenia Malinnikova
Norwegian University of Science and Technology, Trondheim

An improvement of Liouville's theorem for discrete harmonic functions

The classical Liouville theorem says that if a harmonic function on the plane is bounded then it is a constant. At the same time for any angle on the plane, there exist non-constant harmonic functions that are bounded everywhere outside the angle.

The situation is different for discrete harmonic functions on the standard square lattices. The following strong version of the Liouville theorem holds on the two-dimensional lattice. If a discrete harmonic function is bounded on a large portion of the lattice then it is constant. A simple counter-example shows that in higher dimensions such improvement is no longer true. There are some open interesting questions in higher dimensions that we will discuss.

The talk is based on a joint work with L. Buhovsky, A. Logunov and M. Sodin.

Dan Mangoubi
The Hebrew University of Jerusalem

Multiplicity of Eigenvalues for the circular clamped plate problem

A celebrated theorem of C.L. Siegel from 1929 shows that the multiplicity of eigenvalues for the Laplace eigenfunctions on the unit disk is at most two. More precisely, Siegel shows that positive zeros of Bessel functions are transcendental. We study the fourth order clamped plate problem, showing that the multiplicity of eigenvalues is uniformly bounded (by not more than six). Our method is based on Siegel-Shidlovskii theory and new recursion formulas. The talk is based on a joint work with Yuri Lvovski.

Pertti Mattila
University of Helsinki

Some applications of the Fourier transform to Hausdorff dimension

I shall survey some old and recent applications of the Fourier transform to geometric problems on Hausdorff dimension: projections, intersections and distance sets.

Vitali Milman
Tel-Aviv University

Flowers and reciprocity in the theory of convexity

Shahaf Nitzan
Georgia Institute of Technology, Atlanta

Reflections on Landau's density theorem

We survey a collection of results, obtained jointly with A.Olevskii, which reflect different aspects of Landau's density theorem for sampling and interpolating sequences.

Joaquim Ortega-Cerdà
Universitat de Barcelona

A sequence of well-conditioned polynomials

We find an explicit sequence of polynomials of arbitrary degree with small condition number. This solves a problem posed by Michael Shub and Stepen Smale in 1993. This is a joint work together with Carlos Beltran, Ujué Etayo and Jordi Marzo.

Ron Peled
Tel-Aviv University

Fluctuations of random surfaces

Sums of independent, identically distributed random variables are a classical topic in probability theory. As is well known, the sum of n variables of zero mean and variance one is a random variable fluctuating on the scale of $\sqrt{n}$ which, after scaling, converges to the Gaussian distribution. Moreover, the trajectory of all partial sums up to n, properly scaled, converges to a Brownian motion. Such partial sums may be viewed as a function on the discrete segment $\{0, 1, 2, ..., n\}$. In statistical physics and other disciplines, the need arises to consider similarly-defined functions on the d-dimensional domain $\{0, 1, ..., n\}^d$, which may be considered as random (hyper-)surfaces defined on a discrete domain. How do such random surfaces behave: What is the order of their fluctuations and what is their limiting object? To what extent does the behavior depend on specific details of the model (i.e., does the model exhibit universality)? We will give a review intended for non-experts and mention some recent results obtained jointly with Piotr Miłoś, Maxime Gagnebin and Alexander Magazinov.

Zeev Rudnick

Tel-Aviv University

Prime lattice points in ovals

The study of the number of lattice points in dilated regions has a long history, with several outstanding open problems. In this lecture, I will describe a new variant of the problem, in which we study the distribution of lattice points with prime coordinates

We count lattice points in which both coordinates are prime, suitably weighted, which lie in the dilate of a convex planar domain having smooth boundary, with nowhere vanishing curvature.

We obtain an asymptotic formula, with the main term being the area of the dilated domain, and our goal is to study the remainder term. Assuming the Riemann Hypothesis, we give a sharp upper bound, and further assuming that the positive imaginary parts of the zeros of the Riemann zeta functions are linearly independent over the rationals allows us to give a formula for the value distribution function of the properly normalized remainder term.

Eero Saksman

University of Helsinki

Decompositions of log-correlated fields with applications

We consider a simple idea to decompose of log-correlated Gaussian fields into two-parts, both of which behave well in suitable sense. Applications include Onsager type inequalities in all dimensions, analytic dependence and existence of critical chaos measures for a large class of log-correlated fields. Talk is based on Joint work with Janne Junnila (EPFL) and Christian Webb (Aalto University).

Kristian Seip

Norwegian University of Science and Technology, Trondheim

Value distribution of some zeta functions

Zeta functions of the form $\sum_{n=1}^{\infty} \chi(n) n^{-s}$, with χ a completely multiplicative function taking only unimodular values, arise as the limit functions of sequences of vertical translates of the Riemann zeta function in $\operatorname{Re} s > 1$. We study the value distribution of such zeta functions in the half-critical strip $1/2 < \operatorname{Re} s < 1$ in the case when they extend meromorphically at least to the half-plane $\operatorname{Re} s > 1/2$. We give conditions for Voronin universality and show in particular that zeros and poles may be located "anywhere", subject to a density condition akin to the famous density hypothesis, with the zeta function in question nevertheless being universal.

References

[1] K. Seip, *Universality and distribution of zeros and poles of some zeta functions*, `arXiv:1812.11729`.

Boris Solomyak
Bar-Ilan University

Spectral theory of substitution and tiling dynamical systems

Substitution and tiling dynamical systems are of interest, in particular, in the mathematical study of quasicrystals. The dynamical spectrum of such systems is closely related to the diffraction spectrum of the associated atomic structure. We will discuss some new results on fine properties of spectral measures, such as the local dimension and Hoelder continuity. The talk is based on joint work with A. I. Bufetov.

Alexander Ulanovskii
University of Stavanger

Discrete translates in function spaces

For a wide class of separable Banach function spaces X on $\mathbb{R}$, we construct a Schwartz function φ such that the set of translates $\{\varphi(t-\lambda), \lambda \in \Lambda\}$ spans X whenever Λ is an exponentially small perturbation of integers. Essentially, the only exception is the space $L^1(\mathbb{R})$, which cannot be spanned by uniformly discrete translates of a single function. Moreover, there is a pair of Schwartz functions φ_1, φ_2 whose Λ-translates span every "reasonable" Banach function space X, where Λ is as above. Based on joint work with Alexander Olevskii.

James Wright
University of Edinburgh

From Fourier restriction to Diophantine equations (and back)

Recently Bourgain, Demeter and Guth resolved the Vinogradov Mean Value Theorem (a central problem in number theory which counts precisely the number of integer solutions to a classical system of Diophantine equations) as a direct consequence of a basic result in euclidean harmonic analysis (a decoupling theorem). Decoupling theorems underpin many of the central problems in euclidean harmonic analysis such as the Fourier restriction problem. In this talk we will examine the interplay between these two areas at a much more elementary level which will allow us to make connections more seamlessly. The interplay and interactions go in both directions.