Commutative Algebra 88-813

5772 Semester A

Question Sheet 7

Due 5/1/2012, י' טבת תשע"ב

(1) Let F be a field. Recall that if $Z \subset F^n$ is an algebraic set, we can associate to it the ideal $\mathcal{I}(Z) = \{f \in F[x_1, \dots, x_n] : f(a_1, \dots, a_n) = 0 \ \forall (a_1, \dots, a_n) \in Z\}$. Similarly, given an ideal $I \subset F[x_1, \dots, x_n]$, one can associate to it the algebraic set $\mathcal{Z}(I) = \{(a_1, \dots, a_n) \in F^n : f(a_1, \dots, a_n) = 0 \ \forall f \in I\}$.

For any algebraic set $Z \subset F^n$, prove that $Z = \mathcal{Z}(\mathcal{I}(Z))$.

(2) Prove that an algebraic set $Z \subset F^n$ is irreducible if and only if $\mathcal{I}(Z)$ is a prime ideal.

(3) Let R be a Noetherian ring, and let $\varphi : R \to R$ be a ring homomorphism. Prove that if φ is surjective (על), then it is an isomorphism.

(4) Let F be a field. Does the ring extension $F[x] \subset F[x, y]$ satisfy INC?

(5) Let $A \subset R$ be an extension of commutative rings. Let $Q \subset R$ be any ideal. Show that if R is integral over A, then R/Q is integral over $A/(Q \cap A)$.