February 25, 2013

Exam in Modular Forms.

1. Let $\Gamma = SL_2(\mathbb{Z}) \subset SL_2(\mathbb{R}) = G$. Show that the commensurator group $Comm_\Gamma G = SL_2(\mathbb{Q})$.

2. Write explicitly (i.e., using upper triangular representatives of the double co-sets) Hecke operators $\tilde{T}_p$ associated to elements $\begin{pmatrix} p & 0 \\ 0 & p^{-1} \end{pmatrix}$ for p prime.

3. Compute action of $\tilde{T}_p$ on the Fourier expansion $f(q) = \sum_{n>0} a_n q^n$ of holomorphic cusp forms of (even) weight k.

4. What effect the computation in 3) has on the multiplicity one theorem for holomorphic cusp forms?

(The answer is the reason why the classical theory is connected to the group PGL_2 and not to PSL_2.)