Corrections and addenda for Computational Aspects of Polynomial Identities (2005), (AK Peters)

In Proposition 2.44 (Equation (2.5) and in the continuation), the indeterminate x_{n+1} is misplaced, and should instead be placed between x_n and x_k, in order for $\tilde{f}$ to be $(n+1)$-alternating.

Zubrilin's theory, as expounded in §2.5 is incomplete because in the proof of Proposition 2.55, I need not be contained in M of page 63, also cf. Remark 2.50. One can show instead that $I^2 \subseteq M$, yielding $\Phi(A)I^2 \subseteq \Phi(A)M = 0$, but this requires a separate lemma about alternating polynomials from Zubrilin's work based on Theorem 4.82, which is needed to verify Remark 2.42. In other words, Theorem 4.82 should have been given much earlier in order to make the exposition more readible. Full details of the proof are posted on the arXiv:1405.0730 and are incorporated into the revised edition, to appear in 2015.

Thus, p. 65 line 13 should read "$N^{j+2k} \subseteq \Phi(A)I^2 = 0$."

Some more details should be given in the proof of Kemer's representability theorem. Since we are working in characteristic 0, we may assume that all polynomials are multilinear. The proof is based on Wedderburn's decomposition of a finite dimensional algebra as the direct sum of a semisimple subalgebra and the radical, which holds when the base field is perfect, in particular in characteristic 0. We can only assume that each $R_k \cong M_{n_k}(F)$ when F is algebraically closed. The proof in the book makes this assumption on page 96, and the argument of Remark 4.2 enables one to reduce the proof of Specht's conjecture to the algebraically closed case, since $K \subset \bar{F}$ implies K is finite over F, and thus A_0 is finite over F.

The following key observation was used implicitly, since we often need our Kemer polynomials to be full, but was not stated explicitly:

If $\mu > s_A$, then any μ-Kemer polynomial f is full on A.

Proof: Otherwise for any nonzero evaluation of f, any t_A-alternating fold of f has at most $t_A - 1$ semisimple substitutions, and thus at least one radical substitution. Altogether there are at least $\mu > s_A$ radical substitutions, thereby yielding the value 0, a contradiction.

In the proof of Kemer's Second Lemma (Proposition 4.54), the polynomial must realize two properties: Property K and full. To obtain this, let be the T-ideal generated by f. $(\Gamma_1 + \mathrm{id}(A)) \cap \mathrm{id}(\hat{A}_{s-1}) > \mathrm{id}(A)$ since A is PI-subdirectly irreducible. Hence $\Gamma_1 + \mathrm{id}(A)$ contains a nonidentity $\tilde{f} + g$ of A, where $\tilde{f} \in \Gamma_1$ and $g \in \mathrm{id}(\hat{A}_{s-1})$ But then $\tilde{f} = f - g \in \mathrm{id}(\hat{A}_{s-1}) \setminus \mathrm{id}(A)$, and thus is a μ-Kemer polynomial satisfying Property K.

Some readers have expressed concern concerning the assertions on page 122.

Any Kemer polynomial for A_1 must be a Kemer polynomial for the direct product.

The assertion that $\mathrm{id}(\bar{U}) \subseteq \mathcal{S} + \mathrm{id}(\tilde{A})$ is formal, using the fact that each $\bar{U}_i$ is relatively free.

p. 102 Remark 4.17 The bound should be $\beta(W) < s(n^2+1)$ (although this is not needed in the sequel).

p. 103 Insert sentence after Definition 4.21:

We write $\omega(\Gamma)$ for $\omega(W)$ where $\mathrm{id}(W) = \Gamma$.

p. 104 line -12 $\omega(W)\}$ should read $\omega(\Gamma)\}$.

p. 104 line -8 $\Gamma_1 \supseteq \Gamma_2$ should read $\Gamma_1 \subseteq \Gamma_2$.
p. 105 line -11 $X = \{x_{i_1}, \dots, x_{i_t}\}$
p. 108 line 2 identity should read nonidentity
p. 108 line -9 $\oplus_{i,j=1}^{n}$ should read $\oplus_{i,j=1}^{q}$
p. 108 line -6 to q.
p. 109 line -7 Insert: We may assume that A is subdirectly irreducible.
p. 112 Proposition 4.44: We do not need A to be subdirectly irreducible in the hypothesis.
p. 115 The proof of Lemma 4.52 needs clarification. First of all, one can replace A by $\hat{A}_s$, since they have the same identities. Now there is a natural map $\hat{A}_s \to \hat{A}_{s-1}$, which we use to justify the end of the proof.
p. 115 In Lemma 4.53, substitute "full" for Condition (ii).
There are three major items which need further clarification:
p. 125 Theorem 4.66: Throughout, it should be stated explicitly that when taking a relatively free algebra of A, one uses the same number of generators as A. Need reduction to affine, because of Capelli Theorem?
(page 122) A Kemer polynomial of P is a Kemer polynomial of the product?
p. 125
If we apply the Phoenix property to a polynomial with property K then does the consequence have property K?s

Other misprints:
Throughout, Grassman should be Grassmann.
p. 0, line -7,

$$f + \tau f = f(\dots, x_i, \dots, x_j, \dots) + f(\dots, x_j, \dots, x_i, \dots)$$

p. 11, line 11, $f(x_1, \dots, x_t n_1; y_1, \dots, y_m)$
p. 11, line 17, $A = \oplus\{A_m : m \in M\}$,
p. 25, line 7, multilinear $M_n(\mathbb{Q})$-central
p. 25, line 14, erase first expression
p. 26, line -5, in A
p. 27, line 9, Given a K-
p. 27, line -6, $y_1, \dots$
p. ??, line ??, $\Delta(f_j)$
p. 27, line -7, $j \leq m$.
p. 29, line 4, PI-algebra over a field
p. 31, line 7, field (not Noetherian ring)
p. 43, line -4, is a base of A.
p. 46, line 4, $a_1 \cdots a_d = \sum_{\eta \neq 1,\ \eta \in S_d} \alpha_\eta a_{\eta(1)} \cdots a_{\eta(d)}$,
p. 55, line 2 $\tilde{A}/\tilde{J}$
p. 59, line 5 extend it to a module map $A \otimes C\{X\} \to A$ by
p. 60, line -2 x_{n+1}, not zx_{n+1}
all homogeneous polynomials of even (respectively odd) degree p. 63, line 6 a))
p. 63, line 17 Insert: Write $A = C\{a_1, \dots, a_\ell\}$.
p. 63, line 18, 19 a_1 instead of a
p. 63, line -10, -5 $\bar{a}$ instead of $\tilde{a}$
p. 97 line -17 INSERT for $k \neq \ell$,
p. 101, line 5, $\gamma \subseteq \mathrm{id}(\hat{A}')$

l. 7, A should be $\hat{A}$ and A' should be $\hat{A}'$ throughout.
p. 104 line -9 index$(\Gamma_1) \leq$ index(Γ_2).
p. 107 line -17 for each k, $1 \leq k \leq q$.
p. 107 line -11, Conversely, if (4.5) holds,
p. 110 line 9, assume
p. 112 line 2, delete "subdirectly irreducible"
p. 115, line 1, X_s should be X_{s-1}
p. 116, line -6, v_{k_i} should be $v_{k,i}$
p. 116, line -1, $s+1$ should be $s+\mu-1$
p. 117, line 12, $s+u-1$ should be $s+\mu-1$ (twice)
p. 210, line 9, The space given in Lemma 7.21 was initiated by Grishin.