Commutative Algebra 88-813

5772 Semester A

Question Sheet 2

Due 23/11/2011, כ"ו בחשון תשע"ב

(1) Recall that $l(M)$ is the length of a composition series, if it exists, of a module M. If $M = M_1 \oplus M_2$, prove that $l(M) = l(M_1) + l(M_2)$.

(2) Let R be an arbitrary ring, and let M be a simple R-module. Prove that there exists a maximal left ideal $I \subset R$ such that $M \simeq R/I$ as R-modules.

(3) Let F be a field and $n \geq 1$, and let $T_n(F)$ be the ring of all upper triangular $n \times n$ matrices with entries in F. Consider F^n as a $T_n(F)$-module in the obvious way: if $v \in F^n$ and $A \in T_n(F)$, write v as a column and define the scalar multiplication $A \cdot v$ to be the product of matrices Av. Find a composition series for F^n as a $T_n(F)$-module.