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ON MODULAR WEIGHTS OF GALOIS REPRESENTATIONS

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Abstract. Let F be a totally real field and $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ a Galois representation whose restriction to a decomposition group at some place dividing p is irreducible. Suppose that ρ is modular of some weight σ. We specify a set of weights, not containing σ, such that ρ is modular for at least one weight in this set.

Let F be a totally real field and p a prime, and consider a Galois representation $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$. A weight is an irreducible $\overline{\mathbb{F}}_p$-representation of the finite group $\mathrm{GL}_2(\mathcal{O}_F/p)$. There exists a notion of ρ being modular of a certain weight; see, for instance, [1] or [6]. The aim of this note is to prove Theorem 5, which states that if ρ is modular of a weight σ, then it is modular for at least one weight in a certain set of weights distinct from σ. For instance, if p is non-split in F and σ is a quotient of the induction $Ind_{\overline{B}}^{\overline{G}}\theta$ of some character $\theta : \overline{B} \to \overline{\mathbb{F}}_p^*$, where $\overline{B} \subset \overline{G} = \mathrm{GL}_2(\mathcal{O}_K/p)$ is the subgroup of upper triangular matrices, then ρ is also modular for some other Jordan-Hölder constituent of $Ind_{\overline{B}}^{\overline{G}}\theta$.

The earliest version of this statement, in the case $F = \mathbb{Q}$, was proved by Kevin Buzzard but never published. It was written up by Richard Taylor ([7], Lemma 5.1) in the case of p completely split in F and by the author ([5], Corollary 8.4) in general, using a similar argument. The argument here is somewhat more conceptual than the one given in the unpublished [5].

The motivation for proving Theorem 5 is its application to Serre's epsilon conjecture for Hilbert modular forms in [6], where it is stated without proof as Proposition 5.9. If p is unramified in F and ρ is assumed to be modular of some weight, then a conjectural list of the modular weights of ρ was given in [1]. The results towards this conjecture in [4] and [6] both rely on some combinatorial computations which give weaker than desired results for some "irregular" weights; see Remark 5.10 of [6]. Theorem 5 allows us to get around this problem at places of F with small residue field. In particular, one can prove the cases of the epsilon conjecture required to simplify the proof of Serre's conjecture by Khare and Wintenberger by eliminating its reliance on [7]. This is discussed in the remark following Theorem 5.

1. A computation

Let K be a finite extension of $\mathbb{Q}_p$, let π be a uniformizer in K, and denote the residue field $\mathcal{O}_K/(\pi)$ by k. Let $G = \mathrm{GL}_2$, and let $B \subset G$ and $T \subset G$ be the "upper triangular" Borel subgroup and the diagonal torus, respectively. Let $\overline{G} = \mathrm{GL}_2(k)$, and let $\overline{B}$ be the image of $B(\mathcal{O}_K)$ in $\overline{G}$ under the natural projection. Finally, suppose that the (possibly trivial) group N is a finite product $N = \prod_{i=1}^{r} \mathrm{GL}_2(\mathbb{F}_i)$, where the $\mathbb{F}_i$ are finite fields.

Lemma 1. *There exist Shimura varieties $X_1 \to X_0 \xrightarrow{r} X$ over a number field E such that $X_1 \to X$ (resp. $X_1 \to X_0$) is a Galois cover with Galois group $\overline{G} \times N$ (resp. $\overline{B} \times N$).*

Proof. Let E be a totally real field having distinct places $v, w_1, \ldots, w_r$ such that $E_v \simeq K$ and $\mathbb{F}_i$ is the residue field of w_i for each i. Denote $\Sigma = \{v, w_1, \ldots, w_r\}$. Let D/E be a quaternion algebra that splits at exactly one real place of E and at the places in Σ. Note that we allow D to ramify at other finite places, hence there is no condition on the parity of $[E : \mathbb{Q}]$. Let Γ/E be the algebraic group $\mathrm{Res}_{E/\mathbb{Q}} D^*$. Let $\mathbb{A}^\infty$ be the finite adèles of $\mathbb{Q}$. Then

$$\Gamma(\mathbb{A}^\infty) = \prod_{s \in \Sigma} G(E_s) \times \Gamma',$$

where $\Gamma' \subset \Gamma(\mathbb{A}^\infty)$ is supported away from Σ. Let $H \subset \Gamma'$ be an open compact sugroup. If $G^1(\mathcal{O}_{E_v}) \subset G(\mathcal{O}_{E_v})$ is the subgroup of matrices congruent to the identity matrix modulo π, and similarly for the w_i's, then we define open compact subgroups $U, U_0, U_1 \subset \Gamma(\mathbb{A}^\infty)$ as follows:

$$\begin{aligned} U &= G(\mathcal{O}_{E_v}) \times G(\mathcal{O}_{E_{w_1}}) \times \cdots \times G(\mathcal{O}_{E_{w_r}}) \times H \\ U_0 &= B(\mathcal{O}_{E_v}) \times G(\mathcal{O}_{E_{w_1}}) \times \cdots \times G(\mathcal{O}_{E_{w_r}}) \times H \\ U_1 &= G^1(\mathcal{O}_{E_v}) \times G^1(\mathcal{O}_{E_{w_1}}) \times \cdots \times G^1(\mathcal{O}_{E_{w_r}}) \times H \end{aligned}$$

Let X, X_0, and X_1 be the quaternionic Shimura curves over E associated to U, U_0, and U_1 respectively, as in [2]. As in [6], Lemma 3.1, we see that if H is sufficiently small then the maps $X_1 \to X$ and $X_1 \to X_0$ are Galois covers, and the Galois groups clearly are as claimed. □

These Shimura curves carry an action of the group $\Gamma(\mathbb{A}^\infty)$, hence of $G(\mathcal{O}_K)$. If $\eta : \overline{G} \times N \to \mathrm{GL}(V_\eta)$ is any $\overline{\mathbb{F}}_p$-representation of the group $\overline{G} \times N$, then we can define a lisse étale sheaf $\mathcal{F}_\eta$ on X as follows. For any étale cover $Y \to X$, the sections $\mathcal{F}_\eta(Y)$ are functions $F : \pi_0(Y \times_X X_1) \to V_\eta$ such that $F(Cg) = \eta(g)^{-1} F(C)$ for all $g \in \overline{G} \times N$ and all $C \in \pi_0(Y \times_X X_1)$.

Consider the following element $\gamma \in G(K) \subset \Gamma(\mathbb{A}^\infty)$:

$$\gamma = \begin{pmatrix} 1 & 0 \\ 0 & \pi \end{pmatrix}.$$

Then γ determines a Hecke operator $\mathbf{U}_\pi = [U_0 \gamma U_0]$ that acts on the cohomology $H^i(X_0 \otimes \overline{E}, \mathcal{F})$ of any étale sheaf $\mathcal{F}$ on X_0. This action is described, for instance, in [6], section 3.2.

Fix an $\overline{\mathbb{F}}_p$-representation $\xi : N \to \mathrm{GL}(V_\xi)$. It plays no essential role in the argument below, but for applications we need to have Lemma 3 in this generality. Let $\theta : \overline{B} \to \overline{\mathbb{F}}_p^*$ be a character; clearly it factors through $T(k)$. Observe that $W_\theta = \mathrm{Ind}_{\overline{B}}^{\overline{G}} \theta$ can be viewed as the space of functions $f : \overline{G} \to \overline{\mathbb{F}}_p^*$ satisfying $f(bg) = \theta(b) f(g)$ for all $b \in \overline{B}$ and all $g \in \overline{G}$. Consider the étale sheaf $\mathcal{F}_{\theta \otimes \xi}$ on X_0 defined as above: for an étale cover $Y_0 \to X_0$ the sections $\mathcal{F}_{\theta \otimes \xi}(Y_0)$ are functions $G : \pi_0(Y_0 \times_{X_0} X_1) \to \overline{\mathbb{F}}_p \otimes V_\xi$ such that $G(Cb) = \theta(b)^{-1} \xi^{-1}(m) G(C)$ for all $(b, m) \in \overline{B} \times N$ and all $C \in \pi_0(Y_0 \times_{X_0} X_1)$.

Observe that $\mathcal{F}_{W_\theta \otimes \xi} = r_* \mathcal{F}_{\theta \otimes \xi}$. Indeed, an isomorphism is given explicitly as follows. We can evaluate an element of $W_\theta \otimes V_\xi$ at some $g \in \overline{G}$ to obtain an element of $\overline{\mathbb{F}}_p \otimes V_\xi$. Then $F \in \mathcal{F}_{W_\theta\otimes\xi}(Y)$ corresponds to $\tilde{F} \in \mathcal{F}_{\theta\otimes\xi}(Y \times_X X_0)$ if for all $C \in \pi_0(Y \times_X X_1) = \pi_0((Y \times_X X_0) \times_{X_0} X_1)$ we have

$$\tilde{F}(C) = F(C)(I_2),$$

where $I_2 \in \overline{G}$ is the identity matrix. Given $\tilde{F}$, we can recover F by $F(C)(g) = \tilde{F}(Cg^{-1})$ for all $g \in \overline{G}$.

It is a general fact of étale cohomology that if $i \geq 0$, then $H^i(X \otimes \overline{E}, \mathcal{F}_{W_\theta\otimes\xi}) = H^i(X_0 \otimes \overline{E}, \mathcal{F}_{\theta\otimes\xi})$. Hence the operator $\mathbf{U}_\pi$ acts on $H^i(X \otimes \overline{E}, \mathcal{F}_{W_\theta\otimes\xi})$.

Let V' be an $\overline{\mathbb{F}}_p$-vector space on which $\overline{G}$ acts irreducibly with highest weight θ. Let $v \in V'$ be a highest weight vector. We can write a quotient map $\beta : W_\theta \to V'$ explicitly as follows:

$$\beta(f) = \sum_{g\in\overline{B}\backslash\overline{G}} f(g) \cdot g^{-1}v,$$

where in the sum g runs over a set of right coset representatives of $\overline{B}$ in $\overline{G}$. It is easy to verify that this is a non-zero map of $\overline{G}$-modules; indeed, for any $h \in \overline{G}$ we have

$$\beta(hf) = \sum_{g\in\overline{B}\backslash\overline{G}} f(gh) \cdot g^{-1}v = \sum_{g'\in\overline{B}\backslash\overline{G}} f(g') \cdot h(g')^{-1}v = h(\beta(f)).$$

This map induces a map $W_\theta \otimes \xi \to V' \otimes \xi$ of $(\overline{G} \times N)$-modules, and hence a map $\beta : \mathcal{F}_{W_\theta\otimes\xi} \to \mathcal{F}_{V'\otimes\xi}$ of étale sheaves. Over an étale cover $Y \to X$ it is given simply by $\beta(F)(C) = \beta(F(C))$, where the β on the right is our map of $(\overline{G} \times N)$-modules.

We now describe a model for the $\overline{G}$-module V'. The character $\theta : \overline{B} \to \overline{\mathbb{F}}_p^*$ may be expressed in the form

$$\theta : \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \mapsto \prod_{\tau:k\hookrightarrow\overline{\mathbb{F}}_p} \tau(ad)^{w_\tau}\tau(a)^{n_\tau}$$

with $0 \leq w_\tau, n_\tau \leq p-1$. A model for V' is given by the space of polynomials $P(X_\tau, Y_\tau)$ in $2[k : \mathbb{F}_p]$ variables such that P is homogeneous of degree n_τ in the pair X_τ, Y_τ for each $\tau : k \hookrightarrow \overline{\mathbb{F}}_p$. Then an element

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \overline{G}$$

acts by

$$gP(X_\tau, Y_\tau) = \left(\prod_\tau \tau(ad-bc)^{w_\tau}\right) P(\tau(a)X_\tau + \tau(c)Y_\tau, \tau(b)X_\tau + \tau(d)Y_\tau).$$

Clearly $\prod_{\tau:k\hookrightarrow\overline{\mathbb{F}}_p} X_\tau^{n_\tau}$ is a highest weight vector.

Lemma 2. *The group $\overline{G}$ decomposes into $\overline{B}$-cosets as follows:*

$$\overline{G} = \coprod_{a\in k} \overline{B}\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} \coprod \overline{B}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Proposition 3. *There exists an injection of sheaves $\kappa : \mathcal{F}_{V'\otimes\xi} \to \mathcal{F}_{W_\theta\otimes\xi}$ such that the induced map*

$$(\kappa\circ\beta)^* : H^i(X \otimes \overline{E}, \mathcal{F}_{W_\theta\otimes\xi}) \to H^i(X \otimes \overline{E}, \mathcal{F}_{W_\theta\otimes\xi})$$

is equal to the Hecke operator $\mathbf{U}_\pi$.

Proof. To simplify the notation, we suppose that N and ξ are trivial and leave it as a simple exercise for the reader to put the ξ's back in. A vector $v' \in V'$ can be written uniquely in the form $v' = \alpha v + w$, where $\alpha \in \overline{\mathbb{F}}_p$ and w consists of lower weight terms. We write $\langle v', v\rangle = \alpha$. Now $\kappa : \mathcal{F}_{V'} \to r_* \mathcal{F}_\theta$ is defined as follows. Over an étale cover $Y \to X$ we have

$$\begin{array}{rcl} \kappa : \mathcal{F}_{V'}(Y) & \to & \mathcal{F}_\theta(Y \times_X X_0) \\ (\kappa(F))(C) & = & \langle F(C\gamma), v\rangle \end{array}$$

To see that the image indeed lies in $\mathcal{F}_\theta(Y \times_X X_0)$, observe that for all $b \in B(\mathcal{O}_K)$ we have $\gamma^{-1} b \gamma \in G(\mathcal{O}_K)$. Hence for $C \in \pi_0(Y \times_X X_0)$,

$$(\kappa(F))(Cb) = \langle F(Cb\gamma), v\rangle = \langle F(C\gamma \cdot \gamma^{-1} b\gamma), v\rangle = \langle (\gamma^{-1} b^{-1} \gamma) \cdot F(C\gamma), v\rangle.$$

Now observe that $\gamma^{-1} b^{-1} \gamma$ is a lower triangular matrix modulo π. Such matrices can only lower weights but not raise them; if $F(C\gamma) = \alpha v + w$ as above, then $\gamma^{-1} b^{-1} \gamma \cdot w$ has no highest weight component. Hence, from the description of V' above,

$$\langle (\gamma^{-1} b^{-1} \gamma) \cdot F(C\gamma), v\rangle = \langle (\gamma^{-1} b^{-1} \gamma)\alpha v, v\rangle = \theta^{-1}(b)(\kappa(F))(C).$$

This shows that $\kappa(F) \in \mathcal{F}_\theta(Y \times_X X_0)$. Further, if $\tilde{F} \in \mathcal{F}_\theta(Y \times_X X_0)$, then we have

$$(\kappa \circ \beta)\tilde{F}(C) = \langle \beta(\tilde{F})(C\gamma), v\rangle = \langle \sum_{g \in \overline{B} \backslash \overline{G}} \tilde{F}(C\gamma g^{-1}) g^{-1} v, v\rangle.$$

Moreover, it is easy to see from the description of V' above that

$$\begin{array}{rcl} \langle \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}^{-1} v, v\rangle & = & 1 \\ \langle \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{-1} v, v\rangle & = & 0 \end{array}$$

Hence, $(\kappa \circ \beta)\tilde{F}(C) = \sum_{g \in \Delta} \tilde{F}(C\gamma g^{-1}) = \sum_{g \in \Delta} \tilde{F}(C\gamma g)$, where

$$\Delta = \left\{ \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix} : a \in k \right\}.$$

It follows that on cohomology, $(\kappa \circ \beta)^* = \sum_{g \in \Delta} (\gamma g)^*$; here, for $h \in G(K)$, h^* is the operator on cohomology induced by the action of h on X_0. Moreover, there is a coset decomposition

$$U_0 \gamma U_0 = \coprod_{g \in \Delta} \gamma g U_0.$$

By the description of the action of Hecke operators in, for instance, sections 2.3 and 3.2 of [6], we see that $\mathbf{U}_\pi = (\kappa \circ \beta)^*$ as desired.

Finally, observe that κ is injective. Indeed, suppose that $\langle F(C\gamma), v\rangle = 0$ for all $C \in \pi_0(Y \times_X X_1)$ but that $F(C) \neq 0$ for some C. Since V' is generated as a $\overline{G}$-module by the highest weight vector, there exists some $g \in \overline{G}$ such that $g^{-1} \cdot F(C)$ has a highest weight component. Thus $0 \neq \langle F(Cg), v\rangle = (\kappa(F))(Cg\gamma^{-1})$, giving a contradiction. $\square$

2. An application of the computation

Let F be a totally real field. Recall that weights are irreducible $\overline{\mathbb{F}}_p$-representations of $\overline{G} = \mathrm{GL}_2(\mathcal{O}_F/p)$. Even if p ramifies in F, the weights always factor through $\prod_{\mathfrak{p}|p} \mathrm{GL}_2(k_{\mathfrak{p}})$, where $k_{\mathfrak{p}}$ is the residue field of the place $\mathfrak{p}$. Now let $\mathfrak{p}$ be a place of F dividing p and let $K = F_{\mathfrak{p}}$. Put $\overline{G} = \mathrm{GL}_2(k_{\mathfrak{p}})$ and let $\overline{B} \subset \overline{G}$ be the subgroup of upper triangular matrices. Set $N = \prod_{\mathfrak{p}\neq w|p} \mathrm{GL}_2(k_w)$, and we have precisely the setup of the previous section.

In this case we can take X, X_0, and X_1 to be the Shimura curves $M_{0,H}$, $M_{U_0(\mathfrak{p}),H}$, and $M_{1,H}$ respectively, in the notation of [2] and [6]. They are all defined over F. Let χ be the mod p cyclotomic character. From [6], Remark 3.18, we see that $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ is modular of weight σ if and only if $\rho \otimes \chi$ appears in $H^1(X \otimes \overline{F}, \mathcal{F}_\sigma)$. There is a Hecke algebra $\mathbf{T}$ acting on the cohomology. By [6], Proposition 3.17, if ρ is modular, then $\rho \simeq \overline{\rho}_{\mathfrak{m}}$ for a suitable non-Eisenstein maximal ideal $\mathfrak{m} \subset \mathbf{T}$ (see there for the definition of $\overline{\rho}_{\mathfrak{m}}$). Therefore, ρ is modular of weight σ if and only if $H^1(X \otimes \overline{E}, \mathcal{F}_\sigma)_{\mathfrak{m}} \neq 0$.

Consider a character $\theta : \overline{B} \to \overline{\mathbb{F}}_p^*$. The irreducible constituents of $W_\theta = Ind_{\overline{B}}^{\overline{G}}\theta$ are described explicitly in [3], Proposition 1.1. Let V'_θ be the $\overline{\mathbb{F}}_p$-representation of $\mathrm{GL}_2(k_{\mathfrak{p}})$ of highest weight θ. Fix an irreducible $\overline{\mathbb{F}}_p$-representation ξ of N.

Lemma 4. *Suppose that $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ is modular of weight $V'_\theta \otimes \xi$, and let $\mathfrak{m} \subset \mathbf{T}$ be the maximal ideal associated to ρ. Assume that the restriction of ρ to a decomposition group $G_{\mathfrak{p}}$ at $\mathfrak{p}$ is irreducible. Then $H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{\sigma\otimes\xi})_{\mathfrak{m}} \neq 0$ for some constituent σ of $\ker(W_\theta \to V'_\theta)$.*

Proof. Suppose the contrary. Denote by L the kernel of the surjection $W_\theta \to V'_\theta$. Consider the following piece of the long exact cohomology sequence:

$$\begin{array}{ccc} H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{L\otimes\xi})_{\mathfrak{m}} & \to & H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{W_\theta\otimes\xi})_{\mathfrak{m}} \overset{\beta^*}{\to} \\ H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{V'_\theta\otimes\xi})_{\mathfrak{m}} & \to & H^2(M_{0,H} \otimes \overline{F}, \mathcal{F}_{L\otimes\xi})_{\mathfrak{m}} \end{array}$$

The first group is trivial by assumption, and the last one is trivial as $\mathfrak{m}$ is non-Eisenstein ([6], Remark 3.6). Hence β^* is an isomorphism. Since κ^* is injective and $H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{V'_\theta\otimes\xi})_{\mathfrak{m}} \neq 0$, it follows by Proposition 3 that the action of $\mathbf{U}_\pi$ on $H^1(M_{0,H} \otimes \overline{F}, \mathcal{F}_{W_\theta\otimes\xi})_{\mathfrak{m}}$ is non-zero and injective. But it is known that if $\rho|_{G_{\mathfrak{p}}}$ is irreducible, then the Hecke operator $\mathbf{U}_\pi$ does not act as a unit. Indeed, let f be a mod p Hilbert modular form giving rise to ρ as in [6]. If $\mathbf{U}_\pi$ acts as a unit, then f is ordinary; but then $\rho|_{G_{\mathfrak{p}}}$ is reducible by [8], Theorem 2. Hence we get a contradiction. □

Theorem 5. *Let $\theta : \overline{B} \to \overline{\mathbb{F}}_p^*$ be a character, and let $\rho : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$ be such that $\rho|_{G_{\mathfrak{p}}}$ is irreducible and ρ is modular of weight $V'_\theta \otimes \sigma^{\mathfrak{p}}$, where $\sigma^{\mathfrak{p}}$ is a representation of $\prod_{\mathfrak{p}\neq w|p} \mathrm{GL}_2(k_w)$. Then there exists a constituent $\sigma_{\mathfrak{p}} \neq V'_\theta$ of $V = Ind_{\overline{B}}^{\overline{G}}\theta$ such that ρ is modular of weight $\sigma_{\mathfrak{p}} \otimes \sigma^{\mathfrak{p}}$.*

Proof. From [3], Proposition 1.1 we observe that any irreducible representation of $\mathrm{GL}_2(k_{\mathfrak{p}})$ occurs at most once as a Jordan-Hölder constituent of V. The theorem is now immediate from Lemma 4. □

Remark 6. Suppose that $[k_\mathfrak{p} : \mathbb{F}_p] = 2$. Using Theorem 5, one shows in [6], section 5.2 that if $\sigma_\mathfrak{p} \otimes \sigma^\mathfrak{p}$ is a modular weight of ρ, and ρ is locally irreducible at $\mathfrak{p}$, then $\sigma_\mathfrak{p}$ must be in the set $W_\mathfrak{p}(\rho)$ of $\overline{\mathbb{F}}_p$-representations of $\mathrm{GL}_2(k_\mathfrak{p})$ given in section 3 of [1]. Hence, if $[k_v : \mathbb{F}_p] \leq 2$ for all $v|p$, then all modular weights of ρ must be contained in the list of weights conjectured in [1] to be modular. On the other hand, one can show using the methods of [4] that all these weights are indeed modular; thus one has a proof of Serre's epsilon conjecture in this case. As indicated in Taylor's lectures at the summer school on Serre's conjecture at Luminy in July 2007, this is enough to simplify the proof of Serre's conjecture by eliminating its reliance on the elegant, but somewhat complicated, arguments of [7].

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