

Moed Bet Exam 5770 - my answers to Maple questions. Note there are many ways to do everything!

Section A - Question 4

> restart;

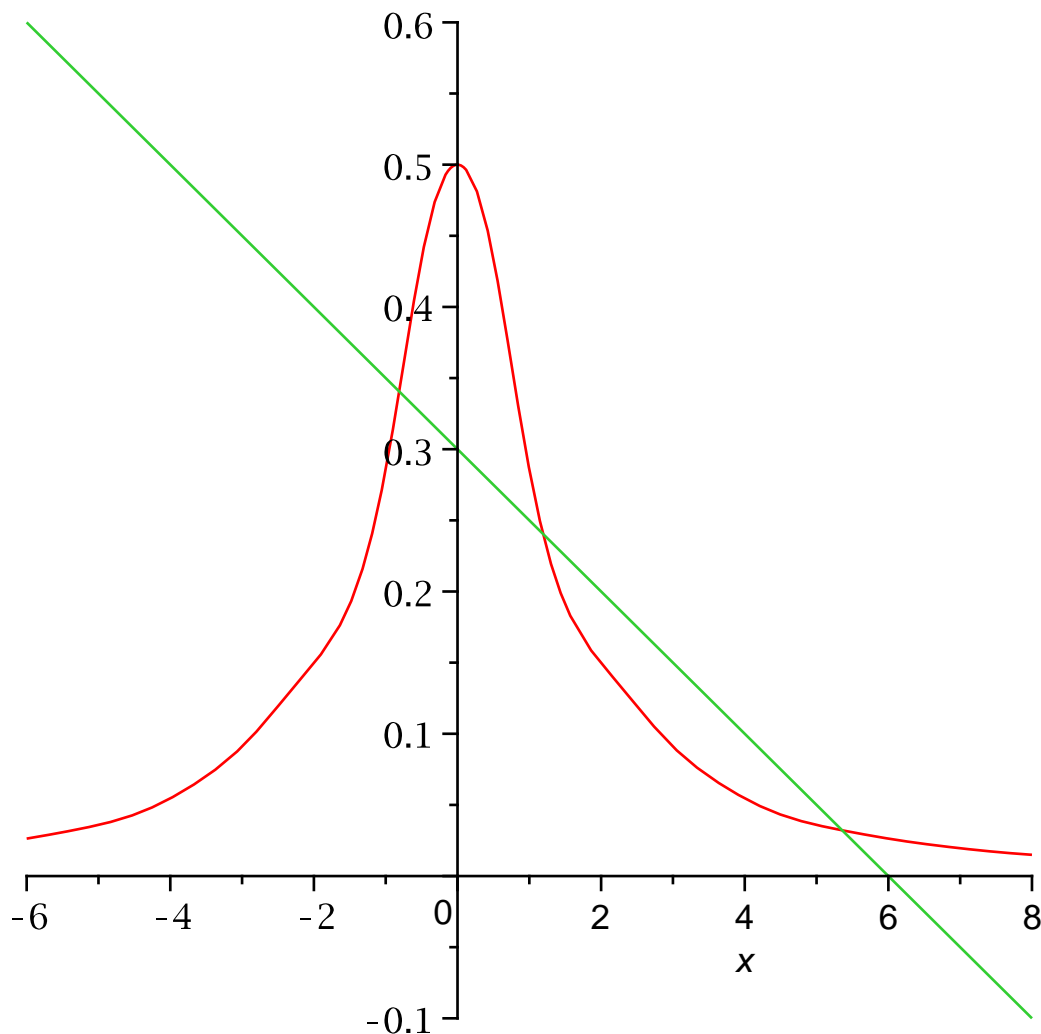
>  $f := \frac{1}{2 + x^2 + \sin(x)^4}; g := \frac{1}{20} \cdot (6 - x);$

$$f := \frac{1}{2 + x^2 + \sin(x)^4}$$

$$g := \frac{3}{10} - \frac{1}{20} x$$

(1)

> plot([f, g], x=-6..8) Not necessary for the question, just showing how I made the plot



>  $r1 := fsolve(f = g, x = -1); r2 := fsolve(f = g, x = 1); r3 := fsolve(f = g, x = 5)$

$$r1 := -0.8117923652$$

$$r2 := 1.189662145$$

$$r3 := 5.357038230$$

(2)

>  $A1 := \text{int}(f - g, x = r1..r2); A2 := \text{int}(g - f, x = r2..r3)$

$$A1 := 0.2476452791$$

$$A2 := 0.1714504879 \quad (3)$$

**Section A - Question 5**

```
> f:=proc(a, b, n)
  local p, i;
  p := a/b;
  for i from 1 to n do
    p := p*(a+i)/(b+i);
  end do;
  return(p);
end proc;
```

```
f:=proc(a, b, n) (4)
```

```
  local p, i;
  p:= a/b; for i to n do p:= p*(a+i)/(b+i) end do; return p
```

```
end proc
```

```
> f(a, b, 8)
```

$$\frac{a(a+1)(a+2)(a+3)(a+4)(a+5)(a+6)(a+7)(a+8)}{b(b+1)(b+2)(b+3)(b+4)(b+5)(b+6)(b+7)(b+8)} \quad (5)$$

```
> f(1/2, 1/3, 10)
```

$$\frac{731794257}{325058560} \quad (6)$$

**Section A - Question 6**

```
> restart; with(LinearAlgebra) :
```

```
> A := <<1, 0, 0|<0, 2, 0|<0, 0, 3>>; B := <<0, 0, 0|<1, 0, 0|<2, 3, 0>>; C := <<0, 1,
```

$$A := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$B := \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

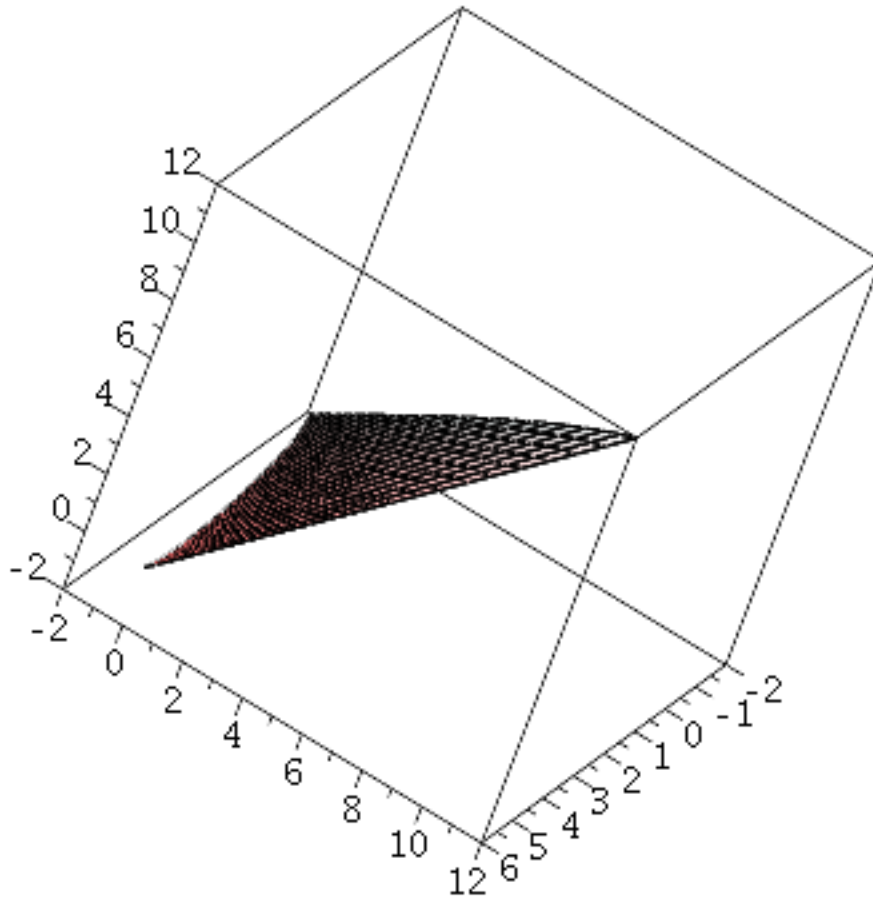
$$C := \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 3 & 0 \end{bmatrix} \quad (7)$$

```
> x := Determinant(A + s*B + t*C); y := Determinant(s*A + B + t*C); z := Determinant(s*A + t*B + C)
```

$$\begin{aligned}
 x &:= 6 - 20st + 6s^2t + 6ts^2 \\
 y &:= 6s^3 - 20st + 6t^2 + 6t \\
 z &:= 6s^3 - 20st + 6t^2 + 6t
 \end{aligned}$$

(8)

> `plot3d( [x, y, z], s = 0..1, t = 0..1, axes = boxed)`



### Section B - Question 1

Part a

I will do the following - assuming B is nonzero I can solve the equation of the line to write  $y = -(Ax+C)/B$ . Then I can put this in the equation of the circle, solve to find the x's (0, 1 or 2) and then get the y's back. If  $B=0$  then  $x=-C/A$  and I solve the equation of the circle to find the y's.

```

> findpts := proc(v)
  local A, B, C, a, b, r, x, ys, y, xs, sols, i;
  A := v[1]; B := v[2]; C := v[3]; a := v[4]; b := v[5]; r := v[6];
  if B = 0 then
    x := -C/A;
    ys := [solve( (x - a)^2 + (y - b)^2 = r^2, y)];
    sols := NULL;
  
```

```

    for i in ys do sols := sols, [x, i] end do:
    return(sols);
else
    y := - (A·x + C) / B;
    xs := [solve( (x - a)2 + (y - b)2 = r2, x)];
    sols := NULL;
    for i in xs do sols := sols, [i, subs(x = i, y)] end do:
    return(sols) :
end if;
end proc:

```

intersection of y – axis and circle center origin, radius 5:

```
> findpts([1, 0, 0, 0, 0, 5])
```

[0, 5], [0, -5] (9)

intersection of x – axis and circle center origin, radius 5

```
> findpts([0, 1, 0, 0, 0, 5])
```

[5, 0], [-5, 0] (10)

Part b

Find the intersection points using part a - if they exist - and then find the minimum distance between a pair of point

Start by making a function to find the distance between 2 points:

```

> dist := proc(a, b)
    return((a[1] - b[1])2 + (a[2] - b[2])2);
end proc:
> finddist := proc(v)
    local p1, p2, d, d2, i, j;
    p1 := [findpts([v[1], v[2], v[3], v[4], v[5], v[6]])];
    p2 := [findpts([v[1], v[2], v[3], v[7], v[8], v[9]])];
    if nops(p1) > 0 and nops(p2) > 0 then
        d := evalf(dist(p1[1], p2[1]));
        for i from 1 to nops(p1) do
            for j from 1 to nops(p2) do
                d2 := evalf(dist(p1[i], p2[j]));
                if d2 < d then d := d2 end if;
            end do;
        end do;
        return(sqrt(d));
    else
        return(infinity);
    end if;
end proc:

```

The need for the "evalf" here is unfortunate - in many cases it will not work without this

Distance on y-axis between circle of radius 1 centered at (0,-3) and circle off radius

1 centered at (0,3) is 4

```
> finddist([1, 0, 0, 0, -3, 1, 0, 3, 1])  
4.000000000
```

(11)

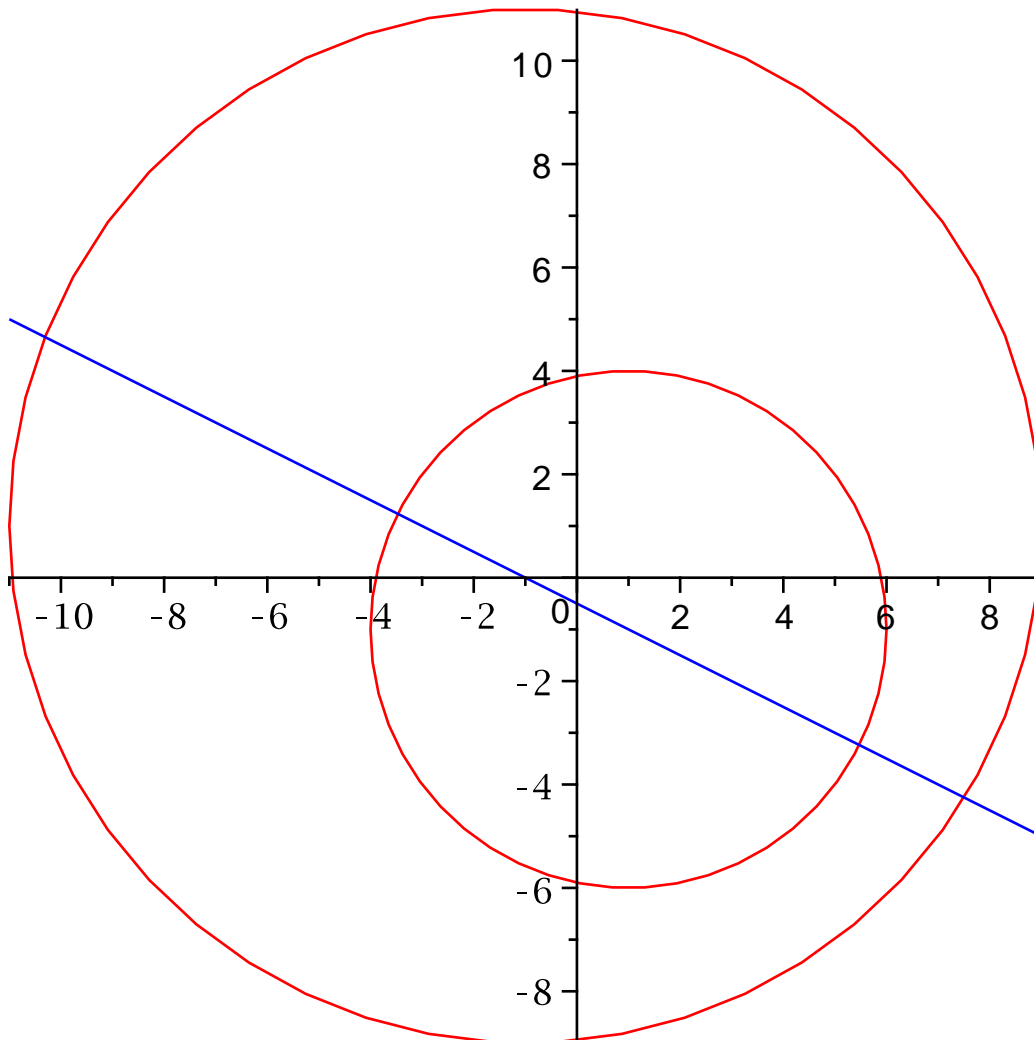
Another example:

```
> finddist([1, 2, 1, 1, -1, 5, -1, 1, 10])  
2.276638107
```

(12)

Drawing of the last situation:

```
> with(plots) : with(plottools) :  
> c1 := circle([1, -1], 5, color = red) : c2 := circle([-1, 1], 10, color = red) : l1  
:= line([-11, 5], [9, -5], color = blue) :  
> display(c1, c2, l1)
```



```
>
```