

Percentage points of student's t_n distribution

Usage: If X_1, \dots, X_n is a sample from $N(\mu, \sigma)$, and

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i, \quad s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2,$$

then

$$\frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

has the t_{n-1} distribution.

The t_n distribution has density

$$\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi n} \Gamma\left(\frac{n}{2}\right) \left(1 + \frac{x^2}{n}\right)^{\frac{n+1}{2}}}.$$

The expectation is 0 (for $n \geq 2$) and the variance $\frac{n}{n-2}$ (for $n \geq 3$).

It follows that we have the following symmetric confidence interval for μ :

$$\mathbf{P}\left(\bar{X} - \mathbf{t} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + \mathbf{t} \frac{s}{\sqrt{n}}\right) = 1 - \alpha$$

where the values \mathbf{t} , which depend on $n - 1$ and α are given as follows:

$n - 1$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	$\alpha = 0.005$	$\alpha = 0.002$	$\alpha = 0.001$
1	6.31	12.7	31.8	63.7	127	318	637
2	2.92	4.30	6.96	9.92	14.1	22.3	31.6
3	2.35	3.18	4.54	5.84	7.45	10.2	12.9
4	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	1.89	2.36	3.00	3.50	4.03	4.79	5.41
8	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	1.81	2.23	2.76	3.17	3.58	4.14	4.59
12	1.78	2.18	2.68	3.05	3.43	3.93	4.32
14	1.76	2.14	2.62	2.98	3.33	3.79	4.14
16	1.75	2.12	2.58	2.92	3.25	3.69	4.02
18	1.73	2.10	2.55	2.88	3.20	3.61	3.92
20	1.72	2.09	2.53	2.85	3.15	3.55	3.85
25	1.71	2.06	2.49	2.79	3.08	3.45	3.73
30	1.70	2.04	2.46	2.75	3.03	3.39	3.65
40	1.68	2.02	2.42	2.70	2.97	3.31	3.55
50	1.68	2.01	2.40	2.68	2.94	3.26	3.50
60	1.67	2.00	2.39	2.66	2.91	3.23	3.46
70	1.67	1.99	2.38	2.65	2.90	3.21	3.44
80	1.66	1.99	2.37	2.64	2.89	3.20	3.42
90	1.66	1.99	2.37	2.63	2.88	3.18	3.40
100	1.66	1.98	2.36	2.63	2.87	3.17	3.39
∞	1.6449	1.9600	2.3263	2.5758	2.8070	3.0902	3.2905

The $n = \infty$ results are given to higher accuracy and correspond to the limiting case of the normal distribution.