General Probability 88-262 **Dr.M.Gorfine, Dr.J.Schiff**

Exam 1, Semester 1, 5762

Exam time: two hours.
You may use any written material you wish and a pocket calculator.
You should answer 7 of the 10 questions. All questions have equal credit.
All answers should be explained fully!

1. a. If I throw a fair die n times, where n is a positive integer, what is the probability I will obtain results that are all different from each other?
   b. If I throw a fair die X times, where X is a random variable with Poisson distribution with parameter λ, what is the probability I will obtain results that are all different from each other?

2. If a random variable X has distribution $B(n,p)$ and has mean 40 and variance 30, what are n and p? Find $\mathbf{P}(|X-40|>10)$.

3. The random variable X has exponential distribution with parameter λ. The random variable Y is defined by
$$Y=\begin{cases}0 & X\le a\\ 1 & a<X<b\\ 2 & X\ge b\end{cases}$$
where a,b are positive reals. Find a,b as functions of λ such that Y has a uniform distribution on $\{0,1,2\}$.

4. In a certain game, the player throws two fair dice. If the sum of the results is 4 or less, the player loses \$72. If the sum is between 5 and 8 the player wins \$36, and if the sum is 9 or more the player wins \$144. What are the mean and variance of the winnings of a player in this game?

5. The random variable X has the following density function:
$$f_X(x)=\begin{cases}a & -1\le x<0\\ \frac{1}{3} & 0\le x<2\\ \frac{1}{4} & 2\le x<3\\ 0 & \text{otherwise}\end{cases}$$
Find the mean and variance of X.

6. In the local grocery store 20% of the rolls are from bakery A and the rest from bakery B. The distribution of the weight (in grammes) of a roll is normal with mean 100 and variance 25 for bakery A, and normal with mean 90 and variance 100 for bakery B.
   a. If a customer chooses a roll in the store at random, what is the probability its weight will be more than 100 grammes?
   b. If a customer chose a roll, and it turns out that the roll weighs more than 100 grammes, what is the probability it is from bakery A?

7. The probabilities of the events A and B are $\mathbf{P}(A) = 0.4$ and $\mathbf{P}(B) = 0.7$ respectively. In each of the following sections briefly explain your answers:
   a. Is it possible that A and B are mutually exclusive?
   b. Is it possible that A and B are independent?
   c. Find the possible values of $\mathbf{P}(A \cap B)$.
   d. Is it possible that $\mathbf{P}(A|B) = 2\mathbf{P}(A)$?
   e. Is it possible that $\mathbf{P}(A|B) = \frac{1}{2}\mathbf{P}(A)$?

8. A ladder has 10 rungs. A person going up the ladder can go up one rung or two with each step. In how many different ways can this person reach the top of the ladder?

9. The joint distribution of the pair of random variables (X, Y) is given in the following table:

		X		
		0	1	2
Y	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
	1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
	2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

   a. Find $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$, $\mathrm{Var}(X - Y)$.
   b. Are X, Y dependent? Explain.
   c. In general, if X and Y are independent random variables, what can be said about $\mathrm{Var}(X - Y)$?

10 The random variable X takes values on the interval $(-1, 1)$ and has density function

$$f_X(x) = \begin{cases} \frac{\pi}{4}\cos\left(\frac{\pi x}{2}\right) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   a. If $0 \leq a < 1$, what is $\mathbf{P}(|X| < a)$?
   b. Find the density function of the random variable $Y = X^2$.

Good luck!