General Probability 88-262 **Dr.M.Gorfine, Dr.J.Schiff**

Exam 2, Semester 1, 5762

Exam time: two hours.

You may use any written material you wish and a pocket calculator.

You should answer 7 of the 10 questions. All questions have equal credit.

All answers should be explained fully!

1. On a package of rice manufactured in a certain factory it is written that the weight of the package is 200 gram. In fact this is not exact, and the real weight is normally distributed with mean 200 gram. Let X be the weight in grams of a randomly selected package of rice.
   a. It is known that $\mathbf{P}(X < 193.3) = 0.25$. What is the standard deviation of the weight of the package?
   b. 100 packages of rice are chosen at random. What is the variance of the number of packages, out of the 100, that weight less than 190 gram?

2. Let X be a continuous random variable with the following density function, depending on a paramter θ:

$$f_X(x) = \begin{cases} \frac{1}{(x-\theta)^2} & x \geq 1+\theta \\ 0 & \text{otherwise} \end{cases}$$

   a. Determine the cummulative distribution function of X, $F_X(x)$.
   b. If $\theta = 0$, what can be said about $\mathbf{E}[X]$?

3. A drunk leaves the bar and tries to get to his home. Due to his lack of sobriety the direction of each step he takes is a random variable, independent of all previous steps, and he steps east with probability p and west with probability $1-p$. Find the probability that after n steps he is r steps east of the entrance to the bar, where r is an arbitrary integer.

4. 80% of the students in a probability course came to the lectures, the others did not. For a student who came to the lectures the chance of passing the exam (each time he is examined) is 85%. For a student who did not attend, the probability is only 50%. Each student does exams until the first time he gets a passing grade. If a certain student passed the exam on his nth attempt (n a positive integer), what is the probability this student did attend the lectures?

5. An electrician buys certain electrical components in packages of 10. The electrician will only buy a package after he has randomly sampled 3 of the components in the package and found all 3 to be functioning properly. It is known that in 30% of the packages there are 4 damaged components, and in the other 70% just 1. What is the probability that the electrician will not buy a package he checks?

6. The random variable X takes the values $\{1,2,3,4\}$ with equal proabability. The random variable Y has distribution $B(1,\frac{1}{3})$. X and Y are independent.
   a. Find the distribution of $X+Y$. What is the variance of $X+Y$?
   b. Find the distribution of XY. What is the variance of XY?

7. On the basis of the information available to them, a certain university divides its incoming students into three categories: excellent (10%), good (40%) and average (50%). The probability a student will reach the second year of studies is 0.9 for an excellent student, 0.8 for a good student and 0.7 for an average student. If a certain student has reached the 2nd year, what is the probability that at the time he was admitted to the university he was in the "excellent" category?

8. A man buys a lottery ticket that gives him the opportunity to participate in 500 independent prize drawings, in each one of which there is a probability $\frac{1}{100}$ to win. Find approximations to the probability that he wins in at least 5 drawings
   a. Using the Poisson approximation to the binomial distribution, and
   b. Using the normal approximation to the binomial distribution.

   Which approximation is better?

9. A and B are arbitrary events. Which of the following are always true? Explain.
   a. $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$
   b. $\mathbf{P}(A \cap B) \leq \mathbf{P}(A)\mathbf{P}(B)$
   c. $\mathbf{P}(A|B) = 1 - \mathbf{P}(A|B^c)$
   d. $\mathbf{P}(A \cup B) = 1 - \mathbf{P}(A^c \cap B^c)$

10. A random variable has the following density function:

$$f_X(x) = \begin{cases} 1 & -a - \frac{1}{2} < x < -a \\ 1 & a < x < a + \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

   where a is a large positive parameter.
   a. Draw the graph of $f(x)$.
   b. Give an intuitive explanation why one might think the standard deviation should be roughly a.
   c. Compute the standard deviation of X.

Good luck!