Conditioning

Let A be the matrix

$$\begin{bmatrix} -1.84 & 18.8 & -3.59 & 1.49 \\ -3.30 & 17.1 & -1.15 & 1.13 \\ -3.06 & 15.5 & -0.907 & 1.05 \\ 3.91 & 0.749 & -5.40 & 1.17 \end{bmatrix}.$$

To three figure accuracy, the solution to

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 9.29 \\ 8.56 \\ 7.79 \\ 0.058 \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.675 \\ 0.669 \\ 0.571 \\ 0.00149 \end{bmatrix}.$$

To three figure accuracy, the solution to

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 9.45 \\ 8.60 \\ 7.91 \\ 0.046 \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3.48 \\ 1.37 \\ 2.65 \\ -0.211 \end{bmatrix}.$$

Denoting the two “input” vectors as b_1 and b_2, i.e. setting

$$b_1 = \begin{bmatrix} 9.29 \\ 8.56 \\ 7.79 \\ 0.058 \end{bmatrix} \qquad b_2 = \begin{bmatrix} 9.45 \\ 8.60 \\ 7.91 \\ 0.046 \end{bmatrix},$$

we have (in the infinity norm):

$$\frac{||b_1 - b_2||}{||b_1||} \approx 0.00286 .$$

Denoting the two “output” vectors as v_1 and v_2, i.e. setting

$$v_1 = \begin{bmatrix} 0.675 \\ 0.669 \\ 0.571 \\ 0.00149 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 3.48 \\ 1.37 \\ 2.65 \\ -0.211 \end{bmatrix},$$

we have (in the infinity norm):

$$\frac{||v_1 - v_2||}{||v_1||} \approx 4.15 .$$

The condition number of the matrix A is approximately 1590.