Question 1 on Targil 2

By Ito's lemma:

$$\begin{aligned} df &= (f_t + af_X + \frac{1}{2}b^2 f_{XX})dt + bf_X dW \\ dg &= (g_t + ag_X + \frac{1}{2}b^2 g_{XX})dt + bg_X dW \end{aligned}$$

and

$$\begin{aligned} d(fg) &= ((fg)_t + a(fg)_X + \frac{1}{2}b^2(fg)_{XX})dt + b(fg)_X dW \\ &= \Big((f_t g + fg_t) + a(f_X g + fg_X) + \frac{1}{2}b^2(f_{XX}g + 2f_X g_X + fg_{XX})\Big)\, dt + b(f_X g + fg_X)dW \\ &= fdg + gdf + b^2 f_X g_X dt \end{aligned}$$

Thus

$$\underline{d(fg) = fdg + gdf + b^2 f_X g_X dt}$$

In greater generality if we try to compute $d(f_1 f_2 \ldots f_n)$ the extra terms arise from the fact that

$$\begin{aligned} (f_1 f_2 \ldots f_n)_{XX} &= (f_1)_{XX} f_2 \ldots f_n + f_1(f_2)_{XX} \ldots f_n + \ldots + f_1 f_2 \ldots (f_n)_{XX} \\ &\quad +2\left(f_{1X} f_{2X} \ldots f_n + \ldots + f_{1X} f_2 \ldots f_{nX} + \ldots + f_1 f_{2X} \ldots f_{nX}\right) \end{aligned}$$

Thus

$$\underline{d(f_1 f_2 \ldots f_n) = \sum_{i=1}^{n} f_1 f_2 \ldots f_{i-1} df_i f_{i+1} \ldots f_n + b^2 dt \sum_{i=1}^{n} \sum_{j=i+1}^{n} f_1 f_2 \ldots f_{i-1} f_{iX} f_{i+1} \ldots f_{j-1} f_{jX} f_{i+1} \ldots f_n}$$

The first term is the usual Leibniz rule. The second term contains a sum of terms obtained by differentiating 2 out of the d functions $f_1, f_2, \ldots, f_n$.

In the case $a = 0$ and $b = 1$ we have $X = W$. Setting $f_1(X,t) = f_2(X,t) = \ldots = f_n(X,t) = X$ we immediately obtain

$$\underline{d(W^n) = nW^{n-1}dW + \frac{1}{2}n(n-1)W^{n-2}dt}$$