APERIODIC ORDER – ASSIGNMENT 3

Due on Thursday, June 11

Exercise 1. (Curtis-Hedlund-Lyndon theorem) Let (X, T) and (Y, T) be two symbolic dynamical systems over $\mathcal{A}$, and suppose that $F : X \to Y$ is a continuous map wich commutes with the shift: $F \circ T = T \circ F$. Prove that F is a sliding block code. (See Lecture 7 summary for the definition.)

Hint: use the fact that a continuous map on a compact space is uniformly continuous, and recall the definition of the metric on symbolic spaces.

Exercise 2. Let $\mathcal{T}$ be a self-similar tiling of the plane with expansion map $\phi = \lambda R_\theta$, where θ/π is irrational. Show that the tiles cannot be polygons.

Hint: use Finite Local Complexity.

Exercise 3. Let ω be a primitive tile substitution. Prove that there exists $n \in \mathbb{N}$ such that ω^n has a fixed point: a tiling $\mathcal{T}$ such that $\omega^n(\mathcal{T}) = \mathcal{T}$.

Exercise 4. Let $\mathcal{T}$ be an FLC fixed point of a tile substitution ω with a *primitive* substitution matrix S_ω. Show that if $\mathbf{0}$ lies in the interior of a $\mathcal{T}$-tile, then $\mathcal{T}$ is repetitive.

Exercise 5. Let ω be a tile substitution which has a fixed point $\mathcal{T}$, that is, $\omega(\mathcal{T}) = \mathcal{T}$. Let $X_{\mathcal{T}}$ be the corresponding tiling space. Prove that the map ω acts on $X_{\mathcal{T}}$ surjectively.