# ***On interactions between noncommutative rings, braces and geometry***

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# Outline

1. Prime radical, definitions

2. New results: Prime radical in rings

3. New results: Prime radical in braces

# Prime rings

The prime radical of a ring is the intersection of all prime ideals in a ring.

A ring is prime if the product of any two non-zero ideals in this ring is non-zero.

An ideal I in a ring R is prime if the factor ring R/I is a prime ring.

# Prime radical

The Prime radical of a noncommutative ring is the intersection of all prime ideals in a ring.

The Prime radical is also called a Baer radical.

A ring is prime radical if it has no prime ideals (except of itself).

Let A be a ring. A differential polynomial ring as a vector space is the ring of polynomials R[x] with multiplication given by:

$$ax=xa+D(a)$$

where D is a derivation of A, so:

$$D(a+b)=D(a)+D(b) \text{ and}$$
$$D(ab)=D(a)b+aD(b).$$

A ring is prime radical if it has no prime ideals (except of itself).

Theorem (Greenfeld, Ziembowski, A.S 2017):

If $R$ is prime radical and $\delta$ is a derivation of $R$, then the differential polynomial ring $R[X; \delta]$ is locally nilpotent.

# New results on Prime radical in rings

# Prime ideals in braces

Louis Rowen mentioned that it would be interesting to investigate prime ideals in braces.

We will say that a brace is prime if the product of any non-zero ideals in this brace is non-zero.

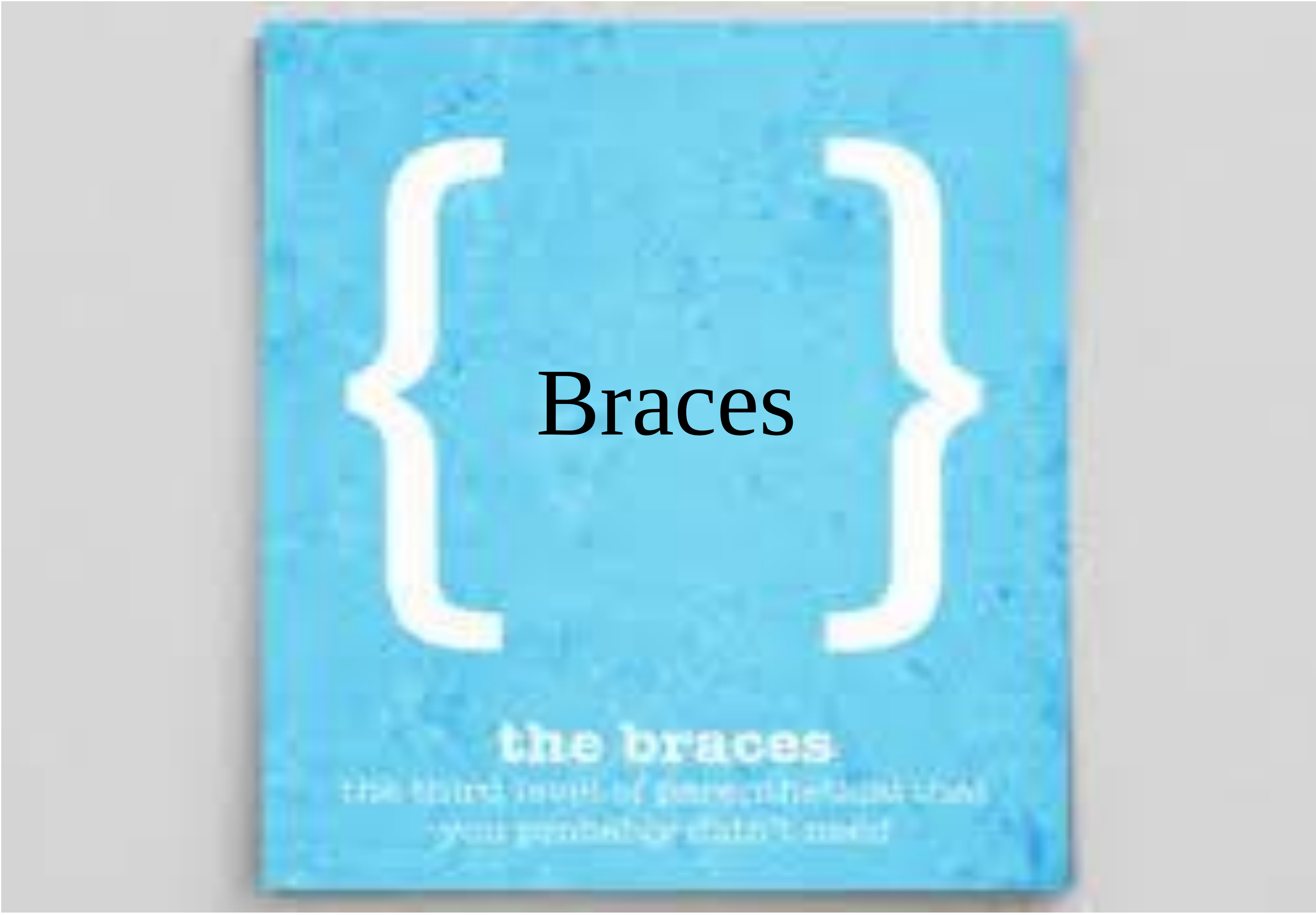
We say that an ideal I in a brace B is prime if B/I is a prime brace.

In 2007, Rump introduced braces as a generalization of Jacobson radical rings related to non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation.

***“With regard to the property that A combines two different equations or groups to a new entity, we call A a brace”***

*Wolfgang Rump*

# Braces



**Definition.** A **left brace** is a set G with two operations + and * such that:

(G,+) is an abelian group

$$\mathbf{a * (b+c) = a * b + a * c}$$

for all a, b, c ∈ G.

(G, ∘) is a group where a∘b=a*b-a-b.

We will say that a brace is **prime** if, whenever I and J are non-zero ideals in A, then the product I*J is non-zero.

We say that a brace A is **semiprime** if, whenever I is a non-zero ideal in A, then the product I*I is non-zero.

We say that an **ideal** I in a brace B is prime (semiprime) if B/I is a prime (semiprime) brace.

# Solvable braces

# Solvable braces

Solvable braces were introduced in a recent paper by Bachiller, Cedo, Jespers and Okninski.

Definition:

Let A be a brace, define

A[1]=A, A[2]=A*A,

A[n+1]=A[n]*A[n],

then A is solvable if **A[k]=0** for some k.

# Solvable braces

Definition(BCJO):

Let A be a brace, define
$A[1]=A$, $A[2]=A*A$,
$A[n+1]=A[n]*A[n]$,
then A is solvable if **$A[k]=0$** for some k.

We say that an ideal I in a brace A is **solvable** if I is a **solvable** brace.

Theorem (Konovalov, Vendramin, A.S., 2018)
A finite brace A is semiprime if and only if A has no solvable ideals.
A semiprime brace can be embedded a direct product of prime braces.
*Remark: The assumption that A is finite is necessary.*

It is easy to show that a brace is semiprime if and only if the intersection of all its prime ideals is zero.

We define the **prime radical** of a brace to be the intersection of all its prime ideals.

Theorem (Konovalov, Vendramin, A.S 2018):

The prime radical of a finite brace equals the  largest solvable ideal of this brace.

Moreover, every brace has the largest solvable ideal which equals the sum of all solvable ideals in this brace.


Very much!