Advnaces in Noncommutative Algebra

On the occasion of Louis Rowen's retirement

Bar Ilan University, June 28th 2017

Abstracts (by order of lectures)

- **Eli Aljadeff** (The Technion) "On the codimension growth of Kemer's fundamental algebras"

  Abstract: In '72 Regev showed the PI codimension sequence $c_n(A)$ is exponentially bounded for any PI algebra A, namely $\limsup(c_n(A)^{1/n}) = d < \infty$. 26 years later Giambruno and Zaicev showed the limit exists and moreover is an integer. In fact, in case the algebra A is finite dimensional, they gave an interpretation of d in terms of the algebra structure. In 2008 Berele and Regev (with an additional result of Giambruno and Zaicev) showed that the codimension sequence $\{c_n(A)\}$ has asymptotically the form $\Theta(n^b d^n)$ where b is half integer. Using Kemer's PI decomposition into fundamental algebras we give an interpretation of the parameter b in case A is finite dimensional. (Joint work with Janssens and Karasik.)

- **Colin Ingalls** (New Brunswich) "Kodaira dimension of Orders"

  Abstract: The Kodaira dimension is the coarsest birational invariant. We present several approaches to the Kodaira dimension of an order and discuss to what extent these are equivalent and invariant under the choice of order. We also show that if we have two division algebra D1 inside D2 with the centres satisfying Z1 inside Z2 Galois, with both finite over their centres, then the Kodaira dimension can only increase. (Joint work with Nathan Grieve.)

- **Murray Schacher** (UCLA) "Norms of integers in central simple algebras"

  Abstract: Let A be a csa over a number field K, R the ring of integers of K. An element of r of R is an outlier if r is the reduced norm of an element of A, but not the norm of an algebraic integer in A. We study the structure and frequency of outliers. We end with a structure theorem about products of super singular elliptic curves.

- **Lenny Makar-Limanov** (Wayne State U) "Freiheitssatz"

  Abstract: Take a group G with n generators and just one relation. Can it be trivial, i.e. are there relators for which $G = 1$? Here we know the answer: $G \neq 1$ and is rather large. Magnus showed in 1930 that such a group contains a free subgroup with $n-1$ generators. This is his famous Freiheitssatz.

  If you are not too familiar with free groups here is a familiar analogy: a linear relation in an n dimensional vector space defines an $n-1$ dimensional subspace.

  Surprisingly enough if A is an associative algebra with n generators and one relation it is still not known whether A is non-trivial. Though it seems obvious that $A \neq 0$ the question is still open when characteristic is not zero.

In my talk I'll discuss possible approaches to this question and tell what is already known.

- **Ed Formanek** (Penn State U) "The Jacobian Conjecture in Two Variables"

  Abstract: Let $\mathbb{C}[x, y]$ be a polynomial ring in two variables over the complex numbers. If $F, G \in \mathbb{C}[x, y]$, then $x \mapsto F$, $y \mapsto G$ determines a $\mathbb{C}$-linear endomorphism of $\mathbb{C}[x, y]$. Associated to a pair (F, G) is a 2×2 Jacobian matrix with entries in $\mathbb{C}[x, y]$. $J(F, G)$ is invertible if and only if $det(J(F, G)) \in \mathbb{C}^*$.

  If $x \mapsto F$, $y \mapsto G$ defines an automorphism of $\mathbb{C}[x, y]$, then the chain rule implies that $J(F, G)$ is invertible. The Jacobian Conjecture in two variables asserts the converse: If $J(F, G)$ is invertible, then $x \mapsto F$, $y \mapsto G$ defines an automorphism of $\mathbb{C}[x, y]$.

  I will survey some of the partial results on the conjecture.

- **Allan Berele** (DePaul) "Invariant theory and trace identities for verbally prime algebras."

  Abstract: Procesi in 1978 used the invariant theory of the general linear group to determine the trace identities of matrices. His methods generalize, so that the invariant theory of the general linear Lie superalgebra yields the trace identities of $M_{k,\ell}$ and the invariant theory of the queer superalgebra yields the queer trace identities of matrices over the Grassmann algebra.

- **Jason Bell** (Waterloo) "The Dixmier-Moeglin equivalence for D-groups"

  Abstract: The Dixmier-Moeglin equivalence is a characterization of the primitive ideals of an algebra that holds for many classes of rings. We investigate a differential-algebraic geometric analogue of this equivalence and show that it holds for a D-groups, which we will define in the talk. We use this to show that the classical Dixmier-Moeglin equivalence holds for Hopf Ore extensions of commutative Hopf algebras. (Joint work with Omar Leon Sanchez and Rahim Moosa.)

- **Susan Montgomery** (USC) "Frobenius-Schur indicators: from groups, through Hopf algebras, to tensor categories"

  Abstract: The definition and many properties of the Frobenius-Schur indicators for the irreducible representations of a finite group were extended to representations of semisimple Hopf algebras in 2000; more recently they have been extended to the objects in well-behaved tensor categories. They are invariants of the category, and determine the category in some nice situations. They have many applications, most recently to von Neumann algebras.

- **Amitai Regev** (Weizmann) "Growth for the central polynomials"

  Abstract: We study the growth of the multi-linear central polynomials for the infinite dimensional Grassmann algebra G, and for the algebra of the k x k matrices, both over a field of characteristic zero. We also review

some new results by Giambruno and Zaicev.

- **Amiram Braun** (Haifa) "On Humphreys' blocks parametrization conjecture"

  Abstract: Let G be a connected, semi simple, simply connected algebraic group over a field of prime characteristic p, and g =Lie(G) its Lie algebra. We shall explain this 1998 conjecture about the blocks of its enveloping algebra U(g), the previously known cases (K.Brown and I.Gordon in 2001) and indicate our solution in case p is "good for G". In particular we shall discuss the new results about the center of U(g) which are crucial in the proof.

- **Miriam Cohen** (Ben Gurion U) "The role of left coideal subalgebra in the structure theory of Hopf algebras"

  Abstract: Left coideal sublagebras are the appropriate analogue in Hopf algebras of subgroups in groups. We discuss their role in Hopf quotients and hence in intrinsic definitions of nilpotency and solvability of Hopf algebras. We'll show how they give rise to Harmonic analysis for Hopf algebras.

- **Jack Sonn** (The Technion) "Quadratic residues, difference sets and Hasse's norm theorem"

  Abstract: Given a prime p, does there exist a subset A of the finite field Fp such that every nonzero quadratic residue modulo p has exactly one representation in the form a"-a' with a",a' in A, and no quadratic non-residue has such representations? We give a number of necessary conditions for this to happen and use them to show that in the range $13 < p < 10^{20}$, there are no primes with the property in question. In the talk we focus on one of the necessary conditions whose proof was discovered by applying Hasse's norm theorem. (Joint work with Seva Lev.)

- **Alexander Merkurjev** (UCLA) "Rationality problem for classifying spaces of algebraic groups"

  Abstract: Many classical objects in algebra such as simple algebras, quadratic forms, algebras with involutions, Cayley algebras etc, can be studied by means of principal homogeneous spaces of linear algebraic groups. Each type of algebraic objects is given by points of the classifying space of an algebraic group. The rationality property for the classifying space yields description of objects by means of algebraically independent parameters. In the talk we will discuss the rationality property of classifying spaces of various algebraic groups.

- **Claudio Procesi** (Universita Di Roma) "Variations on standard identities"

  Abstract: The standard polynomial of degree h is up to a scalar the unique multilinear antisymmetric element of degree h in the free algebra.

When evaluated in an algebra this gives rise to a polynomial map equivariant under the automorphism group. We will investigate a class of problems related to this more general construction.

- **Eva Bayer** (EPFL, Lausanne) "Hasse principle for multinorm equations"
  Abstract: A classical result of Hasse states that the norm principle holds for finite cyclic extensions of global fields, in other words local norms are global norms. We investigate the norm principle for finite dimensional commutative étale algebras over global fields; since such an algebra is a product of separable extensions, this is often called the multinorm principle. Under the assumption that the étale algebra contains a cyclic factor, we give a necessary and sufficient condition for the Hasse principle to hold. (Joint work with Tingyu Lee and Raman Parimala.)

- **Jean-Pierre Tignol** (UC de Louvain) "Around Rowen's crossed product theorem"
  Abstract: In 1978, Louis proved that every division algebra with involution of degree 8 is an elementary abelian crossed product. This talk will present a conceptual proof and discuss some consequences for the construction of invariants of symplectic involutions on central simple algebras of degree 8. (Joint work with Karim Becher and Nicolas Grenier-Boley.)

- **Yoav Segev** (Ben Gurion) "Primitive axial algebras of Jordan type admit a Frobenius form"
  Abstract: Axial algebras (commutative nonassociative) were introduced very recently by Hall, Rehren and Shpectorov, in J. Algebra 437 (2015), 79-115. The notion of primitivity and "f Jordan type" is also introduced there. One of the main motivations was the Monster algebra and the Monster group.
  A Frobenius form on an algebra A is a non-zero bilinear form $(.,.)$ that associates with the algebra product, that is, $(ab, c) = (a, bc)$ for all $a, b, c \in A$. We prove the claim of the title. (Joint work with Jon Hall and Sergey Shpectorov.)

- **Evgeny Shustin** (Tel Aviv) "Refined tropical enumerative invariants"
  Abstract: From the very beginning the tropical mathematics has served as a technical tool for solving problems in the "classical" mathematics. Recently, new objects and ideas born within the purely tropical framework gave rise to new research directions in the classical setting. We demonstrate this phenomenon in the example of refined tropical enumerative invariants, which surprisingly link together some classical complex and real enumerative invariants.

- **Alexei Belov** (Bar Ilan) "Representations of relativelly free algebras and canonization theorems"
  Abstract: The proof of local finite basis property and local representability of relatively free algebras was done, according the Kemer programm. In

order to make the proof more accepteble, following question arises. What does it provide for the community? Why should people read it?

Joint work of speaker with Louis Rowen and Uzi Vishne discovered relations with non-commutative algebraic geometry and provide new insights for representaion theory.

Consider representation ρ of k-algebra A to matrix algebra over algebraic close field K, wich is an affine space. Then Zarissky closer of $\rho(A)$ satysfy the same identities and its natural to investigate representations up to Zarissky clousure wich usually not a linea span if k is finite. $\rho(A)$ decomposes into sum of prime components and Pierce components of radical. First canonization theorem says that it can be reduced to upper block-triangular case. Blocks are either *glued* (may be up to *Frobenious twist*) or independant. Second canonization theorem provides information for quiver or pseudoquiver. Third canonisation theorem says about quiver transformation under factorizing by representable T-ideal. And Fourth (projective) canonization theorem provide existence of non-identities of some canonical structure inside non-zero T-ideal and Phoenix property ("hiking") saying that any element of T-ideal generated by hiked polynomial can be restituted to the same form.

Because one can provide via substitutions on hiked polynomial structure of Notherean module, Specht properties and local representability follows.

- **Be'eri Greenfeld** (Bar Ilan) "Frameworks and Approximations for the Köthe Conjecture"

  Abstract: The Köthe conjecture ("If a ring has a non-zero nil left ideal, then it has a non-zero nil two–sided ideal") has many equivalent formulations, one of them being the following – Suppose R is a nil ring, then R[x] coincides with its own Jacobson radical. Although the conjecture is in general open, and seems quite inaccessible at the moment, much work was done in order to "approximate" it, namely, proving slightly weaker assertions, or disproving slightly stronger ones.

  In this talk we focus on two frameworks which turn out to be valuable from the point of view of such approximations, namely: graded-nil algebras and differential polynomial rings. We present both positive and negative approximations and mark some promising directions for further research. (Based on joint work with J.P. Bell and with A. Smoktunowicz and M. Ziembowski.)

- **Zinovy Reichstein** (Vancouver) "Fields of definition for representations of finite groups."

  Abstract: A classical theorem of Brauer asserts that every finite-dimensional non-modular representation ρ of a finite group G defined over a field K, whose character takes values in a subfield k, descends to k, provided that k has suitable roots of unity. If k does not contain these roots of unity, it is natural to ask how far ρ is from being definable over k.

  he classical answer is given by the Schur index of ρ, which is the smallest degree of a finite field extension l/k such that ρ can be defined over l. In this talk, based on joint work with Nikita Karpenko, Julia Pevtsova and

Dave Benson, I will discuss another invariant, the essential dimension of ρ, which measures "how far" ρ is from being definable over k in a different way, by using transcendental, rather than algebraic field extensions. This invariant is of interest in both the modular and the non-modular settings.

- **Don Passman** (U Wisconsin-Madison) "Logic festival"

  Abstract: The Axiom of Choice has numerous applications in Algebra. For efficient and innovative proofs, it is good to be aware of the many equivalent versions of this axiom, like Zorn's Lemma, The Tychonoff Product Theorem and the Well Ordering Principle. Also of interest are certain associated constructs like Ultra Filters and Ultra Products. Examples will be offered. In particular, we will discuss an application of Ultra Products to the solution of the semiprimitivity problem for group rings of locally finite groups.

- **Andrei Rapinchuk** (Virginia) "On the notion of genus for division algebras and algebraic groups"

  Abstract: Let D be a central division algebra of degree n over a field K. One defines the genus gen(D) of D as the set of classes [D'] in the Brauer group Br(K) where D' is a central division K-algebra of degree n having the same isomorphism classes of maximal subfields as D. I will review the results on gen(D) obtained in the last several years, in particular the finiteness theorem for gen(D) when K is finitely generated of characteristic not dividing n. I will then discuss how the notion of genus can be extended to arbitrary absolutely almost simple algebraic K-groups using maximal K-tori in place of maximal subfields, and report on some recent progress in this direction. (Joint work with V. Chernousov and I. Rapinchuk)