# ABSTRACTS
## (BY ORDER OF THE LECTURES IN THE PROGRAM)

Israel Mathematical Union — American Mathemacial Society The Second International Joint Meeting, June 2014

**List of sessions**:

(1) Geometry and Dynamics
(2) Dynamics and Number Theory
(3) Combinatorics
(4) Financial Mathematics
(5) Mirror Symmetry and Representation Theory
(6) Nonlinear Analysis and Optimization
(7) Qualitative and Analytic Theory of ODE's
(8) Quasigroups, Loops and application
(9) Algebraic Groups, Division Algebras and Galois Cohomology
(10) Asymptotic Geometric Analysis
(11) Recent trends in History and Philosophy of Mathematics
(12) History of Mathematics
(13) Random Matrix Theory
(14) Field Arithmetic
(15) Additive Number Theory
(16) Teaching with Mathematical Habits in Mind
(17) Combinatorial Games
(19) Applications of Algebra to Cryptography
(20) Topological Graph Theory and Map Symmetry
(21) The Mathematics of Menahem M. Schiffer
(22) PDEs: Modeling Theory and Numberics
(23) Contributed Papers (misc. topics)

## Invited Speakers

**Michael Larsen** (Indiana University). *Borel's Theorem on word maps and some recent variants.*
Abstract. Armand Borel proved that if $w$ is a non-trivial word in the free group on generators $x, y, \ldots, z$ and $G$ is a complex semisimple Lie group, then for most elements $g$ of $G$, the equation $w(x, y, \ldots, z) = g$ has a solution in G. This can be thought of as a strong version of the claim that $G$ contains non-commutative free subgroups. There have been a number of recent attempts to strengthen or generalize Borel's theorem. I will sketch Borel's proof and report on recent progress and open problems.

**Tamar Ziegler** (Hebrew U and Technion). *Patterns in primes and dynamics on nilmanifolds.*
Abstract. I will survey some of the ideas behind the recent developments in additive number theory and ergodic theory leading to the proof of Hardy-Littlewood type estimates for the number of prime solutions to systems of linear equations of finite complexity.

**Andrei Okounkov** (Columbia University). *The M-theory index.*
Abstract. I will report on a joint work in progress with Nikita Nekrasov. Our goal is to define the M-theory index for manifolds that fiber in a 5-dimensional (complex) Calabi-Yau variety X over a circle. After an introduction to the problem, I will explain how much progress we made so far and how this problem is related to the Donaldson-Thomas theory of threefolds.

**Gil Kalai** (Hebrew University). *Influences, thresholds, and noise sensitivity.*
Abstract. We will consider the notions of influence and noise sensitivity of Boolean functions and discuss the connection with harmonic analysis, applications, and extensions.
The influence of a variable (or a set of variables) on a function is the probability that changing the value of the variable(s) can change the value of the function. The noise-sensitivity of a function is the probability that for a random assignment to the variables adding a random independent noise will change the value of the function.
We will look at some old and new results and open problems, and mention applications to sharp threshold phenomena, percolation, random graphs, voting, and computation: classic and quantum.
The new results that I will present are based on joint works with Jeff Kahn, Jean Bourgain, and Elchanan Mossel, and on a work in progress with Guy Kindler.

**Eran Nevo** (Ben-Gurion University). *Around the g-conjecture (IMU Erdos Prize lecture).*

Abstract. A major open problem in algebraic combinatorics is McMullen's 1970 g-conjecture, which suggests a characterization of the face numbers of triangulated spheres, of any fixed dimension. The simplicial polytopes case was proved by Billera–Lee (sufficiency) and Stanley (necessity) in 1980, known as the g-theorem. I will discuss some results and problems brunching from this conjecture, from old to very recent: for more general objects (e.g. triangulated manifolds, doubly Cohen-Macaulay complexes), special cases (e.g. PL-spheres, flag spheres), and related objects (e.g. cubical polytopes, complexes embedded in spheres).

**Leonid Polterovich** (Tel-Aviv University). *Symplectic topology: from dynamcis to quantization.*
Abstract. I review a number of advances in function theory on symplectic manifolds and discuss their links to Hamiltonian dynamics and quantum mechanics.

**Ian Agol** (UC Berkeley). 3-*manifolds and cube complexes.*
Abstract. A We will discuss the proof of the virtual Haken conjecture, stating that aspherical 3-manifolds have finite-sheeted covers that are Haken. In fact, we will discuss a conjecture of Dani Wise in geometric group theory which implies this conjecture, as well as related conjectures such as Thurston's virtual fibering conjecture. Part of the results are based on joint work with Daniel Groves and Jason Manning.

**Mihalis Dafermos** (Princeton University). *The mathematics of black hole spacetimes in general relativity.*
Abstract. Black holes constitute one of the most spectacular predictions of general relativity. Some of the very basic questions, however, concerning the dynamics of the Einstein equations in a neighbourhood of the celebrated black hole solutions of Schwarzschild and Kerr remain to this day unanswered. In this talk, I shall describe some recent advances in our understanding of the stability of these spacetimes as well as of the fate of physical observers who fall inside their black hole regions.

## 1. Geometry and Dynamics

**Michael Brandenbursky** (CRM-ISM, University of Montreal). *Concordance group and stable commutator length in braid groups.*
Abstract. In this talk I will define quasi-homomorphisms from braid groups to the concordance group of knots and examine its properties and consequences of its existence. In particular, I will provide a relation between the stable four ball genus in the concordance group and the stable commutator length in braid groups, and produce examples of infinite families of concordance classes of knots with uniformly bounded four ball genus. I will

also provide applications to the geometry of the infinite braid group. In particular, I will show that its commutator subgroup admits a stably unbounded conjugation invariant norm. This answers an open problem posed by Burago, Ivanov and Polterovich. If time permits I will describe an interesting connection between the concordance group of knots and number theory. This work is partially joint with Jarek Kedra.

**Alden Walker** (University of Chicago). *Schottky semigroups.*
Abstract. Let the function f on the complex numbers be multiplication by the complex number z, where z lies in the unit disk. So f is a contracting similarity with center 0 and dilation z. Let g be a contracting similarity with center 1 and dilation z. Consider the semigroup G generated by f and g. Though the definition is quite elementary, many properties of G depend in very subtle ways on the parameter z. For example, we could ask whether the limit set of G is disconnected (in which case we say that G is Schottky), or whether it contains the point 1/2. The study of these questions (and others) turns out to be quite interesting. The primary purpose of this talk is to show many pictures, including a picture proving that the space of Schottky semigroups is disconnected. Joint work with Danny Calegari and Sarah Koch.

**Sheel Ganatra** (Stanford University). *$S^1$-equivariant symplectic homology and cyclic homology.*
Abstract. There is a natural circle action in symplectic homology coming from rotation of free loops; the resulting equivariant theory is related to contact homology and has been studied by Viterbo and Bourgeois-Oancea. There is a second, more algebraic, circle action on cyclically comparable sequences of chords between Lagrangians. We prove that the geometric open-closed string map between these two complexes is $S^1$-equivariant at a suitable chain level. In particular, there are induced maps between associated equivariant homology theories, intertwining Gysin sequences, which are isomorphisms whenever the non-equivariant map is.

**Rafal Komendarczyk** (Tulane University). *Knot and link invariants for vector fields.*
Abstract. In 1979, V. I. Arnold showed that the fundamental invariant of 2–component links i.e. the linking number, can be generalized to an invariant of volume preserving vector fields. In this talk, the Arnold's construction will be outlined, as well as its various applications. Further, more recent results concerning generalizations of this construction to Vassiliev invariants of knots, will be discussed (joint work with Ismar Volic).

**Yoel Groman** (Hebrew University). *Open Gromov Witten invariants of toric Calabi Yau manifolds.*
Abstract. The Gromov Witten invariants of a symplectic manifold M count

pseudo holomorphic curves in M representing a given homology class and subject to appropriate incidence conditions. Analagously, physicists have predicted the existence of open Gromov Witten invariants. For a symplectic manifold M with Lagrangian submanifold L, these should count pseudo holomorphic curves with boundary in L. There are various technical difficulties, most importantly bubbling in codimension one, which prevent the general definition of these invariants. I will discuss a construction of such invariants for a certain class of non compact special Lagrangians in toric Calabi Yau manifolds. Joint work with M. Liu and J. Solomon.

**Vladimir Rovenski** (Haifa University). *Total mixed scalar curvature of foliated Riemannian manifolds.*

Abstract. The problem of minimizing geometric quantities has been very popular since long time. In the context of foliations, Gluck and Ziller (1986) considered the problem of minimizing functions like volume, total energy, and bending defined for $k$-plane fields on Riemannian manifolds. Our main goal is to study Riemannian structures minimizing "total mixed scalar curvature" for adapted variations of metrics on a foliation. Foliations are decompositions of manifolds into collections of submanifolds. Many models in mechanics and globally hyperbolic spacetimes in physics are foliated. The *mixed scalar curvature* $\mathrm{S}_{\,\mathrm{mix}}$ of a foliated manifold $(M^{n+p}, \mathcal{F}^p, g)$ is the averaged mixed sectional curvature (a plane containing a vector tangent to $\mathcal{F}$ and a vector orthogonal to $\mathcal{F}$ is said to be mixed). One may recover Einstein's field equations as the Euler-Lagrange equations of the total scalar curvature. We study similar variational problem related to the total $\mathrm{S}_{\,\mathrm{mix}}$ of arbitrary foliation. We calculate directional derivatives of $J_{\mathrm{mix},\,\Omega}(g) = \int_\Omega \mathrm{S}_{\,\mathrm{mix}}(g)\,\mathrm{d}\,\mathrm{vol_g}$, where $\Omega$ is any relatively compact domain in $M$, characterize critical metrics for some geometrical classes of foliations (see E. Barletta, S. Dragomir, and V. Rovenski, Total mixed scalar curvature of foliations, preprint, 24 pp.) and discuss possibility of applications to physics (see E. Barletta, S. Dragomir, V. Rovenski, and M. Soret, *Mixed gravitational field equations on globally hyperbolic spacetimes*, Class. Quantum Grav., 30, 2013).

**Marc McLean** (Institute for Advanced Study, Princeton). *Minimal Discrepancy of Isolated Singularities and Reeb Orbits.*

Abstract. Let A be an affine variety inside a complex N dimensional vector space which either has an isolated singularity at the origin or is smooth at the origin. The intersection of A with a very small sphere turns out to be a contact manifold called the link of A. Any contact manifold contactomorphic to the link of A is said to be Milnor fillable by A. If the first Chern class of our link is 0 then we can assign an invariant of our singularity called the minimal discrepancy, which is an important invariant in birational geometry. We relate the minimal discrepancy with indices of certain Reeb orbits on our link. As a result we show that the standard contact 5 dimensional

sphere has a unique Milnor filling up to normalization.

**Frol Zapolsky** (Haifa University). *Multiplicity of orbits of contact flows.*
Abstract. I will present an application of the technique of generating functions to the problem of existence and multiplicity of orbits of contact flows with Legendrian boundary conditions. The lecture is going to be elementary and I'll explain all the concepts and techniques necessary for its understanding.

## 2. Dynamics and Number Theory

**Zeev Rudnick** (Tel-Aviv University). *Some problems in analytic number theory for polynomials over a finite field.*
Abstract. I will discuss several problems of analytic number theory in the context of function fields over a finite field, where they can be approached by methods different than those of traditional analytic number theory. The resulting theorems can be used to check existing conjectures over the integers, and to generate new ones. Among the problems discussed are: Counting primes in short intervals and in arithmetic progressions; Chowla's conjecture on the autocorrelation of the Möbius function; and the additive divisor problem.

**Dubi Kelmer** (Boston College). *Equidistribution of translated unbounded geodesics and counting integer points.*
Abstract. I will describe new results on the equidistribution of translated unbounded geodesics on a hyperbolic surface and its application to counting integer solutions to certain Diophantine equations. Joint work with A. Kontorovich.

**Andreas Strömbergsson** (Uppsala University, Sweden). *Effective Ratner equidistribution for* $\mathrm{SL}(2,\mathbb{R}) \ltimes (\mathbb{R}^2)^k$ *and applications to quadratic forms.*
Abstract. Let $G = \mathrm{SL}(2,\mathbb{R}) \ltimes (\mathbb{R}^2)^k$ and let $\Gamma$ be a congruence subgroup of $\mathrm{SL}(2,\mathbb{Z}) \ltimes (\mathbb{Z}^2)^k$. I will present a result giving effective equidistribution of 1-dimensional unipotent orbits in the homogeneous space $\Gamma\backslash G$. The proof involves spectral analysis and use of Weil's bound on Kloosterman sums. I will also discuss applications to effective results for variants of the Oppenheim conjecture on the density of $Q(\mathbb{Z}^n)$ on the real line, where $Q$ is an irrational indefinite quadratic form. Joint work with Pankaj Vishe.

**Mike Hochman** (Hebrew University). *Self-similar sets with overlaps, in* $\mathbb{R}$ *and* $\mathbb{R}^d$.
Abstract. I will describe results concerning the dimension of self-similar sets and measures on $\mathbb{R}$ and $\mathbb{R}^d$. The gist is that the dimension should be

large unless either there are algebraic relations between the generating similitudes, or there is a linear subspace which intersects the set (or measure) in a maximal way. As time permits I will say a little but about how additive combinatorics plays a role.

**Andre Reznikov** (Bar-Ilan University). *On the conformal type of the Dirichlet's unit lattice of a number field.*
Abstract. The celebrated Dirichlet's unit theorem associates (via the logarithmic embedding) to a number field a lattice in an appropriate euclidean space. We discuss conjectures concerning shapes of these lattices (as sublattices in the euclidean space) as the number field varies in various natural families. The same object appears naturally in dynamics/geometry as conformal types of totally flat tori in spaces of the type $\mathrm{SL}(n,\mathbb{Z})\mathrm{SL}(n,\mathbb{R})$ (and in such context was suggested by M. Gromov).

**Jeremy Kahn** (CUNY Grad Center). *Surface Subgroups for Subgroups of* $\mathrm{PSL}_2(\mathbb{C})$.
Abstract. After a brief review of my construction (with V. Markovic) of surface subgroups of discrete cocompact subgrups of $\mathrm{PSL}_2(\mathbb{C})$, I will attempt to describe the construction of surface subgroups of some indiscrete subgroups of $\mathrm{PSL}_2(\mathbb{C})$, such as $\mathrm{PSL}_2$ of the ring of integers of a number field, following ideas of Danny Calegari and Vladimir Markovic.

**Emmanuel Breuillard** (Universite Paris-Sud, France). *Uniform spectral gaps and random matrix products.*
Abstract. Zariski-dense subgroups of a semisimple algebraic group are uniformly non amenable, i.e. the trivial representation is uniformly isolated in the spectrum of their regular representation. I will explain how to use the theory of random matrix products to extend this to quasi-regular representations on Schreier graphs, where the isotropy group is not Zariski dense. This can be viewed as a refinement of Borel's density theorem to thin groups. From this one can also derive explicit lower bounds on the spectral gap in the super-strong approximation theorem.

**Nicolas De Saxce** (Hebrew University). *Expansion in simple Lie groups.*
Abstract. Given a finite set $A$ in a group $G$, one can consider the product set $AAA$ of elements of $G$ that can be written as product of three elements of $A$. If the ambient group $G$ is a compact simple Lie group, a result of Breuillard and Green shows that if $|AAA| < K|A|$, then $A$ must be included in a proper closed subgroup of $G$ as soon as $|A|$ is large enough. I will explain how to make sense of this kind of statement when $A$ is an infinite set, and discuss applications to the study of Borel measurable subgroups of a simple Lie group (starting with $\mathrm{SU}(2)$).

**Barak Weiss** (Tel-Aviv University). *Badly approximable vectors on fractals and random walks.*

Abstract. In joint work with David Simmons, we prove that certain natural fractal measures arising from iterated function systems of contracting similarities, give zero measure to badly approximable vectors. Previous results in this direction involved a reduction to the measure rigidity results of Lindenstrauss. Our approach involved a reduction to the analysis of stationary measures for certain semigroups acting on homogeneous spaces, extending work of Benoist and Quint.

## 3. Combinatorics

**Van Vu** (Yale University). *Real roots of random polynomials.*

Abstract. Consider a polynomial $P_n = c_0 + c_1 x + \cdots + c_n x^n$ of degree n whose coefficients $c_i$ are iid real random variables with mean 0. In late 1930s and eary 1940s, Littlewood and Offord surprised the mathematics community by showing that the number of real roots of $P_n$ is quite small, somewhere between $\log n / \log\log n$ and $\log^2 n$. Subsequent deep works of Littlewood-Offord, Erdos-Offord, Turan, Kac, Stevens, Ibragimov-Maslova and others showed that the expectation of the number of real roots is $2/\pi \log n + o(\log n)$. In the case when $c_i$ are gaussian, Wilkins, (and also Edelman and Kostlan several years later) showed that the error term $o(\log n)$ is actually $O(1)$, giving a very precise result. Nothing close to this has been known for other variables (such as $\pm 1$). The current best estimate from Ibragimov and Maslova's paper is $O(\log^{1/2} n \log\log n)$. We are going to present a new approach to the studies of roots of random polynomials (complex and real alike). As a consequence, we can show that the error term is $O(1)$ (the same as is in the Gaussian case) for general non-Gaussian random polynomials.

**Oriol Serra** (Universitat Politecnica de Catalunya, Barcelona). *Arithmetic Removal Lemmas.*

Abstract. The combinatorial removal lemma, that a graph with no many copies of a given graph H can be made H-free by removing a small number of edges, has been formulated in an arithmetic version by Ben Green. It says that a linear equation in an abelian group with not too many solutions in a given set A with linear size, has no solutions in a dense subset of A. I will discuss several extensions to this result to nonabelian groups and to linear systems. These use the hypergraph removal lemma. Applications to arithmetic Ramsey theory, including counting versions of monochormatic solutions to linear systems or a Roth theorem for 3-term arithmetic progressions in nonabelian groups, will be also discussed. Joint work with Dan Kral' and Lluis Vena.

**Tali Kaufman-Halman** (Bar-Ilan University). *Bounded degree high dimensional expanders.*
Abstract. Gromov and independently Linial and Meshulam suggested two related notions of high dimensional expanders. Gromov defined "topological overlapping" while Linial and Meshulam defined "coboundary expansion". A major open problem (asked by Gromov and others) is whether *bounded degree* high dimensional expanders according to these definitions exist for $d \geq 2$. We show, for the first time, bounded degree complexes of dimension $d = 2$ which have the topological overlapping property. Assuming a conjecture of Serre on the congruence subgroup property, these complexes are also coboundary expanders. Joint work with David Kazhdan and Alex Lubotzky.

**Rom Pinchasi** (Technion). *On the union of arithmetic progressions.*
Abstract. We show that for every $\epsilon > 0$ there is an absolute constant $c(\epsilon) > 0$ such that the following is true: The union of any n arithmetic progressions, each of length n, with pairwise distinct differences must consist of at least $c(\epsilon)n^{2-\epsilon}$ elements. We show also that this type of bound is essentially best possible, as we observe n arithmetic progressions, each of length n, with pairwise distinct differences such that the cardinality of their union is $o(n^2)$. Joint work with Shoni Gilboa.

**Isabella Novik** (University of Washington, Seattle). *Face numbers of balanced spheres, manifolds, and pseudomanifolds.*
Abstract. A simplicial $(d-1)$-dimensional complex $\Delta$ is called balanced if the vertices of $\Delta$ can be colored in $d$ colors in such a way that no two adjacent vertices are assigned the same color. In the talk we will discuss several recent results and conjectures on the face numbers of balanced spheres, manifolds, and pseudomanifolds, among them the balanced analog of the Lower Bound Theorem (including the treatment of equality) and the balanced analog of the Generalized Lower Bound Conjecture. Joint work with Steve Klee.

**Edward Scheinerman** (Johns Hopkins University, Baltimore). *On Vertex, Edge, and Vertex-Edge Random Graphs.*
Abstract. We consider three classes of random graphs: edge random graphs, vertex random graphs, and vertex-edge random graphs. Edge random graphs are Erdos-Renyi random graphs, vertex random graphs are generalizations of geometric random graphs, and vertex-edge random graphs generalize both. The names of these three types of random graphs describe where the randomness in the models lies: in the edges, in the vertices, or in both. We show that vertex-edge random graphs, ostensibly the most general of the three models, can be approximated arbitrarily closely by vertex random graphs, but that the two categories are distinct. Joint work with Elizabeth

Beer, James Fill, and Svante Janson.

**Yael Tauman Kalai** (MSR, New England). *The Evolution of Proofs in Computer Science.*
Abstract. One of the most fascinating developments of theoretical computer science is the evolution of proofs: zero-knowledge proofs, interactive proofs, multi-prover interactive proofs, and probabilistically checkable proofs. In this talk I will give a (too) brief overview of this evolution, and show how these notions can be used to solve the "real world" problem of delegating (or outsourcing) computation.

**Irit Dinur** (Weitzman Institute). *Lifting locally consistent solutions to global solutions.*
Abstract. We are given a collection of (alleged) partial views of a function. We are promised "local consisency", i.e., that the partial views agree on their intersection with probability p. The question is whether the partial views can be *lifted* to a global function f, i.e. whether a p' fraction of the partial views agree with f (aka "global consistency").
This scenario captures "low degree tests" and "direct product tests", both studied for constructions of PCPs. We describe other possible settings where a lifting theorem may hold.
We are interested in understanding the limiting parameters for such lifting theorems. We describe open problems and conjectures whose solution can lead to proving the "sliding scale conjecture", in particular, constructing PCPs with polynomially small soundness error.

**Benny Sudakov** (ETH, Zurich). *The minimum number of nonnegative edges in hypergraphs.*
Abstract. Given an $r$-uniform $n$-vertex hypergraph $H$ it is easy to see that one can assign weights to its vertices with zero sum, such that the number of edges whose total weight is nonnegative is at most the minimum degree of $H$ (e.g., put weight $n-1$ on the minimum degree vertex and $-1$ everywhere else).
In this talk we show that this estimate is tight for every weightings of $H$ which has equal codegrees. As immediate corollaries this implies two conjecture posed by Manickam-Miklos-Singhi in late 80's. Joint work with Hao Huang.

## 4. Financial Mathematics

**Jean-Pierre Fouque** (University of California Santa Barbara). *Mean Field Games and Systemic Risk.*
Abstract. We propose a simple model of inter-bank borrowing and lending where the evolution of the log-monetary reserves of N banks is described by

a system of diffusion processes coupled through their drifts in such a way that stability of the system depends on the rate of inter-bank borrowing and lending. Systemic risk is characterized by a large number of banks reaching a default threshold by a given time horizon. Our model incorporates a game feature where each bank controls its rate of borrowing/lending to a central bank. The optimization reflects the desire of each bank to borrow from the central bank when its monetary reserve falls below a critical level or lend if it rises above this critical level which is chosen here as the average monetary reserve. Borrowing from or lending to the central bank is also subject to a quadratic cost at a rate which can be fixed by the regulator. We solve explicitly for Nash equilibria with finitely many players, and we show that in this model the central bank acts as a clearing house, adding liquidity to the system without affecting its systemic risk. We also study the corresponding Mean Field Game in the limit of large number of banks in the presence of a common noise. Joint work with R. Carmona and L.-H. Sun.

**Andreaa Minca** (Cornell University). *Systemic Risk with Central Counterparty Clearing.*
Abstract. This paper studies financial networks in a stochastic framework. We measure systemic risk in terms of a risk adjusted valuation principle. The framework allows us to examine the effects on systemic risk and price contagion of multilateral clearing via a central clearing counterparty (CCP). We prove existence and uniqueness of an interbank payment equilibrium in conjunction with the price impact on external assets. We find that a CCP not always reduces systemic risk and provide sufficient conditions for the latter to hold. We derive the capitalization of a CCP based on game theoretic arguments. A real world calibrated numerical study illustrates our findings. Joint work with H. Amini and D. Filipovic.

**Mete Soner** (ETH Zurich). *Rebalancing the portfolio.*
Abstract. In markets with friction, such as transaction costs, illiquidity, it is very costly to follow a desired optimal portfolio. This necessitates rebalancing rules that keeps the actual portfolio process close to the desired one but without accumulating large costs due to frictions. In this talk, we assume that the optimal portfolio is obtained as a solution of a frictionless utility maximization problem. We then use methods of partial differential equations and homogenization techniques to obtain asymptotic formulae. These results provide asymptotically optimal rebalancing rules and allows us to compute several quantities of interests.

**Alexander Schied** (University of Mannhein, Germany). *A hot-potato game under transient price impact.*
Abstract. We consider a Nash equilibrium between two high-frequency traders in a simple market impact model with transient price impact and additional quadratic transaction costs. We show that for small transaction

costs the high-frequency traders engage in a "hot-potato game", in which the same asset position is sold back and forth. We then identify a critical value for the size of the transaction costs above which all oscillations disappear and strategies become buy-only or sell-only. Numerical simulations show that for both traders the expected costs can be lower with transaction costs than without. Moreover, the costs can increase with the trading frequency when there are no transaction costs, but decrease with the trading frequency when transaction costs are sufficiently high.

**Mathieu Rosenbaum** (University Pierre and Marie Curie (Paris 6)). *Limit theorems for nearly unstable Hawkes processes.*
Abstract. Because of their tractability and their natural interpretations in term of market quantities, Hawkes processes are nowadays widely used in high frequency finance. However, in practice, the statistical estimation results seem to show that very often, only nearly unstable Hawkes processes are able to fit the data properly. By nearly unstable, we mean that the L1 norm of their kernel is close to unity. We study in this work such processes for which the stability condition is almost violated. Our main result states that after suitable rescaling, they asymptotically behave like integrated Cox Ingersoll Ross models. Thus, modeling financial order flows as nearly unstable Hawkes processes may be a good way to reproduce both their high and low frequency stylized facts. We then extend this result to the Hawkes based price model introduced by Bacry et al. We show that under a similar criticality condition, this process converges to a Heston model. Again, we recover well known stylized facts of prices, both at the microstructure level and at the macroscopic scale. Joint work with Thibault Jaisson (Ecole Polytechnique Paris).

**Michael Ludkovski** (University of California Santa Barbara). *Optimal Execution under Dynamic Order Flow Imbalance.*
Abstract. We analyze an optimal execution model that takes into account the dynamics of market and limit order flows. The trader's footprint on the order flow results in an information leakage that is modeled as a permanent adverse selection cost. In addition, there is the temporary price impact from consuming liquidity. Trading is done in volume-time enabling direct comparison to the VWAP benchmark. We model the market order imbalance with an Ornstein-Uhlenbeck process and formulate a finite-fuel control problem which under certain assumptions, has a closed form solution. We also discuss closed-form approximations to the adaptive execution horizon problem, providing a dynamic extension of the one-period "Optimal Execution Horizon" model of Easley et al. (2012). Numerical examples and discussion of the relevant empirical data will also be given. Joint work with K. Bechler (UCSB).

**Yan Dolinsky** (Hebrew University). *Martingale Optimal Transport in the Skorokhod Space.*
Abstract. The dual representation of the martingale optimal transport problem in the Skorokhod space of multi dimensional cadlag processes is proved. The dual is a minimization problem with constraints involving stochastic integrals and is similar to the Kantarovic dual of the standard optimal transport problem. The constraints are required to hold for very path in the Skorokhod space. This problem has the financial interpretation as the robust hedging of path dependent European options. Joint work with H.M Soner.

**Mathias Beiglbock** (University of Vienna). *Model-Independent Finance, Optimal Transport and Skorokhod Embedding.*
Abstract. Model-independent pricing has grown into an independent field in Mathematical Finance during the last 15 years. A driving inspiration in this area has been the fruitful connection to the Skorokhod embedding problem. We discuss a more recent approach to model-independent pricing, based on a link to Monge-Kantorovich optimal transport. This transport-viewpoint also sheds new light on Skorokhod's classical problem.

**Antoine Jacquier** (Imperial College London). *Small-maturity asymptotics for the forward implied volatility.*
Abstract. Asymptotics of implied volatility have been the subject of active research over the past decade. We look here at a slightly different problem, namely that of the small-maturity behaviour of the forward implied volatility (that is the implied volatility corresponding to forward-start options). In the context of stochastic volatility models, this can be studied, mathematically, as a small-time problem for diffusions, albeit with random initial data.
This initial randomness makes this problem more subtle to handle, and we show that the forward implied volatility 'explodes (with a precise rate) as the remaining maturity tends to zero. This in particular sheds new light on the use of stochastic volatility models for the dynamics of implied volatility.

**Ariel Neufeld** (ETH Zurich). *Superreplication under Volatility Uncertainty for Measurable Claims.*
Abstract. We establish the duality formula for the superreplication price in a setting of volatility uncertainty which includes the example of random G-expectation. In contrast to previous results, the contingent claim is not assumed to be quasi-continuous. Joint work with Marcel Nutz.

**Eilon Solan** (Tel-Aviv University). *Equilibria in Stopping Games with Stopping Rates.*
Abstract. Multiplayer stopping game with termination rates are continuous-time stopping games in which when some players stop, the game does not

terminate with probability 1, but rather stops with some rate, that may depend on time and on the set of players who stop at that time.
We present the model of stopping games with termination rates and prove that every such game admits an $\epsilon$-equilibrium, for every $\epsilon > 0$.

**Gadi Fibich** (Tel-Aviv University). *Is Heterogeneity Important?.*
Abstract. Typically, a model with a heterogeneous property is considerably harder to analyze than the corresponding homogeneous model. In this talk I will show that any outcome of a heterogeneous model that satisfies the two properties of differentiability and symmetry is $O(\epsilon^2)$ equivalent to the outcome of the corresponding homogeneous model, where $\epsilon$ is the level of heterogeneity. In such cases, therefore, the effect of heterogeneity is minor. Applications of this "averaging principle" to open problems in queuing theory, game theory (auctions), and marketing (diffusion of new products in social networks) will be presente.

## 5. Mirror Symmetry and Representation Theory

**Jake Solomon** (Hebrew University). *Open Gromov-Witten theory in dimensions 0, 2 and 3.*
Abstract. I will sketch two different geometric constructions of open Gromov-Witten theory. The first construction applies in dimensions 2 and 3. The second applies in dimension 0. Despite the difference in constructions, the theory in all dimensions fits into a uniform algebraic structure

**Yuval Flicker** (Ariel U and Ohio State U). *Counting local systems with principal local monodromy.*
Abstract. I will report on joint work with Pierre Deligne on counting local systems on open curves over finite fields, with principal unipotent monodromy at infinity. This we achieve on translating via Lafforgue to counting automorphic representations, and further on using the trace formula on the suitable division algebra.

**Michael McBreen** (MIT). *Hypertoric Mirror Symmetry.*
Abstract. Hypertoric varieties are hyperkahler analogues of toric varieties. They arise as the simplest examples of symplectic resolutions, a class of spaces important in representation theory. I will discuss aspects of mirror symmetry for hypertoric varieties. This includes joint work with Daniel Shenfeld on their quantum cohomology, and work in progress on homological mirror symmetry.

**Andrea Appel** (University of Southern California). *Quasi-Coxeter Categories for Kac-Moody Algebras.*
Abstract. In 2005, V. Toledano Laredo proved that the monodromy of the

Casimir connection of a simple Lie algebra g is described by the quantum Weyl group operators of the quantum group $U_h(\mathfrak{g})$. His proof relies upon the notion of a quasi-Coxeter quasitriangular quasibialgebra, which informally is a bialgebra carrying actions of a given generalized braid group and Artin's braid groups on the tensor products of its modules. In this talk, I will give a brief overview of the strategy to extend these results when $\mathfrak{g}$ is an arbitrary symmetrizable Kac-Moody algebra, based upon a generalization of the notion of quasi-Coxeter algebra at a categorical level. Joint work with V. Toledano Laredo.

**Mikhail Mazin** (Kansas State University). *Combinatorics of Affine Springer Fibers.*
Abstract. I will talk about topology and combinatorics of a certain class of type A affine Springer fibers. Let (m,n) be a pair of positive relatively prime integers. We say that a linear subspace M of the ring of formal power series is (m,n)-invariant if it is invariant under multiplication by $t^n$ and $t^m$. The variety $J_m n$ is then defined to be the space of flags of (m,n)-invariant subspaces. I will describe an affine paving of $J_m n$ with cells enumerated by the elements of the affine symmetric group with no inversions of height m, and show that the dimension of the corresponding cell equals to the total number of inversions of height less then m. I will also explain how this is related to combinatorics of parking functions.

**Oren Ben-Bassat** (Haifa U and Oxford U). *Shifted Symplectic Geometry.*
Abstract. I will give an introduction to derived algebraic geometry and in particular the shifted symplectic geometry of Pantev, Toen, Vaquie, and Vezzosi (PTVV). Following work of Toen, Vezzosi and Lurie, derived algebraic geometry is becoming an indispensable language and framework to discuss moduli spaces in algebraic geometry. It also plays a substantial role in the current formulation of the geometric Langlands correspondence and even in theoretical physics for instance in Costello's work on classical field theory, the BV formalism, and supersymmetric gauge theory. I will discuss their example of the shifted symplectic structure on BG where G is an algebraic group. I will also talk about their theorem that the fiber product or "intersection" of two Lagrangians in a shifted symplectic space (derived stack) itself carries ashifted symplectic structure. If time permits, I will discuss some local structure theorems (or Darboux Theorems) of Joyce, Brav and Bussi which can be found at and an interesting structure I found on multiple fiber products.

**Ivan Loseu** (Northeastern University). *Representation theory of quantized quiver varieties.*
Abstract. Nakajima quiver varieties are of great importance in Algebraic Geometry and Geometric Representation theory. Their quantizations are also of representation theoretic interests. I will describe the recent progress

on the representation theory of these quantizations.

**Inna Entova-Aizenbud** (MIT). *Schur-Weyl duality in complex rank.*
Abstract. Let $V$ be a finite dimensional vector space. The classical Schur-Weyl duality describes the relation between the action of the Lie algebra $gl(V)$ and the symmetric group $S_n$ on the n-th tensor power of V. We will discuss Deligne categories $\mathrm{Rep}(S_t)$, which are extrapolations to complex t of the categories of finite dimensional representations of the symmetric groups. I will then present a generalization of the classical Schur-Weyl duality in the setting of Deligne categories, which involves a construction of a complex tensor power of $V$, and gives us a duality between the Deligne category and a Serre quotient of a parabolic category $O$ for $gl(V)$.

**Rami Aizenbud** (Weizmann Institute). *Representation count, rational singularities of deformation varieties, and pushforward of smooth measures.*
Abstract. We will present the following 3 results: 1. The number of n-dimensional irreducible representations of the pro-finite group $\mathrm{SL}(d, \mathbb{Z}_p)$ is bounded by a polynomial on n whose degree does not depend on d and p (our current bound for the degree is 22). 2. Let $\phi : X \to Y$ be a flat map of smooth algebraic varieties over a local field $F$ of characteristic 0 and assume that all the fibers of $\phi$ are of rational singularities. Then, the push-forward of any smooth compactly supported measure on $X$ has continuous density. 3. Let $X = \mathrm{Hom}(\pi_1(S), \mathrm{SL}_d)$ where $S$ is a surface of high enough genus (our current bound for the genus is 12). Then $X$ is of rational singularities.¡br¿We will also discuss the surprising relation between those results which allowed us to prove them.

**Ran Tessler** (Hebrew University). *Intersection Theory on the Moduli of Disks and an Open Analogue for Witten's KdV Conjecture.*
Abstract. We define the moduli space of stable marked discs, its tautological line bundles their gravitational descendants. We then state a theorem which determines all the descendants, and state a conjecture regarding the higher genera generalization. Joint work with R. Pandharipande and J. Solomon.

**Andrei Negut** (Columbia University). *The elliptic Hall algebra and stable basis of the Hilbert scheme.*
Abstract. We will discuss the K-theoretic stable basis (defined by Maulik-Okounkov) in the special case of the Hilbert scheme, and show how it interacts with geometric operators arising from the elliptic Hall algebra. The answer will be a generalization of the Murnaghan-Nakayama rule.

**Galyna Dobrovolska** (Columbia University). *Finite local systems in the Drinfeld-Laumon construction.*
Abstract. Motivated, on the one hand, by R. Bezrukavnikov's conjectures in the representation theory of the rational Cherednik algebra in characteristic

p, and, on the other hand, by geometric Langlands duality for the trivial local system, we calculate explicitly the result of the Drinfeld-Laumon construction applied to the direct summands in the Springer-Laumon sheaf for the trivial local system on a curve which correspond to irreducible representations of the symmetric group (the proof is completed for a curve of genus 0 and in progress for curves of positive genus).

## 6. Nonlinear Analysis and Optimization

**Adi Ben-Israel** (Rutgers University). *Inverse Dynamic Programming.*
Abstract. Let $V(x,k)$ be the optimal (say maximal) value function (OVF) of a $k$-stage dynamic program (DP), given the initial state (or input) $x$. DP is solved by recursion on the OVF, using Bellman's optimality principle. Given a return $v$, a natural question is to find the minimal state $x$ that guarantees that return (e.g., the minimal budget $x$ needed to pave a road of length $v$). The optimal input $x$ corresponds to a return $v$ by means of a function $x := I(v,k)$, called here the optimal input function, (OIF). In principle, DP can also be solved through the OIF, but its recursion is less obvious. The dual relations between optimal value and optimal input were were studied by Iwamoto (J. Math. Anal. Appl., 1977) and others.
They are applied here to DP's with vector states, where the primal solution (using the OVF) is limited by the "curse of dimensionality". An application to the minimum time problem of optimal control is to find how far can one get away from the target and still make it there in a given time. Joint work with T. Asamo.

**Simeon Reich** (Technion). *A Denjoy-Wolff Theorem for Compact Holomorphic Mappings in Complex Banach Spaces.*
Abstract. We establish a Denjoy-Wolff theorem for compact holomorphic self-mappings of bounded and strictly convex domains in arbitrary complex Banach spaces. Joint work with Monika Budzynska and Tadeusz Kuczumow.

**Edriss Titi** (Weizmann Inst and U of California - Irvine). *Finite Number of Determining Parameters for the Navier-Stokes Equations with Applications into Feedback Control and Data Assimilations.*
Abstract. In this talk we will implement the notion of finite number of determining parameters for the long-time dynamics of the Navier-Stokes equations (NSE), such as determining modes, nodes, volume elements, and other determining interpolants, to design finite-dimensional feedback control for stabilizing their solutions. The same approach is found to be applicable for data assimilations. In addition, we will show that the long-time dynamics of the NSE can be imbedded in an infinite-dimensional dynamical system that is induced by an ordinary differential equations, named determining form,

which is governed by a globally Lipschitz vector field. The NSE are used as an illustrative example, and all the above mentioned results hold also to other dissipative evolution PDEs.

**Roman Polyak** (Technion). *Nonlinear Input-Output Equilibrium.*
Abstract. The Nonlinear Equilibrium for Leontiefs input output model was introduced and analyzed. It requires replacing the price and demand vectors by price and demand operators. The conditions on both operators, under which the Nonlinear InputOutput Equilibrium (NIOE) exists and unique were found. Finding NIOE is equivalent to solving a particular variation inequality on a simple feasible set, projection on which require at most matrix by vector multiplication. Therefore we consider two methods: Pseudo Gradient Projection (PGP) and Extra Pseudo Gradient (EPG), for which the projection on the feasible set is the main operation at each step. For both the PGP and the EPG methods the convergence, the global Q-linear rate and the complexity bounds were established under various condition on the input date. The NIOE is a fundamental departure from the classical Leontiefs model. Both methods can be viewed as pricing mechanisms for establishing NIOE in a Productive Economy.

**Roger Nussbaum** (Rutgers University). *Analyticity and Nonanalyticity of Solutions of Delay-Differential Equations.*
Abstract. It is a very special case of an old result of Nussbaum that if x is a bounded $C^1$ function which maps $\mathbb{R}$ into $\mathbb{R}^n$ and solves an autonomous, analytic differential-delay equation with constant time lags, then $x$ is necessarily real analytic. However, the techniques of proof do not necessarily carry over if the time lags are "state-dependent" (e.g., if the time lags are analytic functions of $x(t)$) or if the equations are nonautonomous and the time lags are analytic functions of $t$. We shall describe some new positive results concerning analyticity of bounded solutions of non-autonomous functional differential equations, but we shall also discuss a class of simple-looking linear, non-autonomous differential-delay equations which have infinitely differentiable periodic solutions which are analytic on a nonempty open set but fail to be real analytic at an uncountable set of points $S$. We shall mention a number of open questions concerning the issue of analyticity versus non-analyticity, e.g., the case of periodic solutions of nonlinear, autonomous functional differential equations with state dependent time lags. The results of this lecture represent joint work with John Mallet-Paret. Details can be found in arXiv: 1305.0579v2 (to appear in SIAM J. Mathematical Analysis) and in papers under preparation.

**Hector Sussmann** (Rutgers University). *A Very Weak Regularity Result for Minimizers of Real Analytic Subriemannian Metrics.*
Abstract. Using various results on subanalytic stratifications, we prove that, for a real analytic subriemannian structure, every length-minimizer

parametrized by arc-length is real analytic on an open dense subset of its interval of definition. This is a stronger version of our previous result which, however, is valid for a much larger class of real analytic optimal control problems according to which for every optimal trajectory there exists a possibly different optimal trajectory which is real analytic on an open dense subset of its interval of definition and has the same initial and terminal points.

**Cyril Tintarev** (Uppsala University, Sweden). *A notion of Weak Convergence in Metric Spaces.*
Abstract. The talk introduces a mode of convergence in general metric space ("polar convergence"), which is weaker than standard convergence. The polar limit of a sequence is uniquely defined in uniformly convex metric spaces, and in particular in uniformly convex Banach spaces. In Hilbert spaces polar convergence coincides with weak convergence. We present a proof to the following statement: **Theorem**. Every bounded sequence in a uniformly convex complete metric space has a polar-convergent subsequence.
The argument fits on two slides and does not invoke the Axiom of Choice. We discuss applications of polar convergence to fixed point theory and calculus of variations, in particular, we give a version of Brezis-Lieb Lemma that makes no assumption of a.e. convergence. Joint work with Sergio Solimini.

**Igor Zelenko** (Texas A&M University). *Jacobi Equations and Comparison Theorems in Optimal Control Problems.*
Abstract. We will describe the construction of intrinsic (coordinate-free) Jabobi equations and state-feedback invariants which are responsible for conjugate points along extremals of quite wide class of smooth control system and give the estimates for the number of conjugate points in terms of these invariants. The main difference of this approach compared to the classical ones is that it does not require a construction of a canonical connection at the very first step (the latter is available only in the few classical cases e.g. Riemannian geometry and sub-Riemannian structures associated with Sasakian and more general contact metric structures). Instead, the space of Jacobi fields and the condition for conjugate point along an extremal can be described in terms of certain curve in Lagrangian Grassmannians so that the aforementioned intrinsic Jacobi equation and state-feedback invariants can be deduced from studying the geometry of such curves with respect to the natural action of linear symplectic group.

**Gershon Wolansky** (Technion). *Projection of Gradient Flow of Entropy Like Functionals on a Space of Empirical Measures.*
Abstract. We propose a grid free method for reduction of gradient flow in measure space onto a finite dimensional space of atomic measures. This is done by gamma limit approximation of the entropy onto atomic measures and proving the convergence of the associated gradient descent flow.

**Alexander J. Zaslavski** (Technion). *Structure of Approximate Solutions of Optimal Control Problems.*
Abstract. We study the turnpike property of approximate solutions of optimal control problems considered on subintervals of a real line. To have this property means, roughly speaking, that the approximate solutions of the problems are determined mainly by the integrand (objective function), and are essentially independent of the choice of interval and endpoint conditions, except in regions close to the endpoints. We consider a class of optimal control problems (with the same system of differential equations, the same functional constraints and the same boundary conditions) which is identified with the corresponding complete metric space of objective functions (integrands) and establish the turnpike property for any element of a set which is a countable intersection of open everywhere dense sets in the space of integrands. This means that the turnpike property holds for most optimal control problems (integrands).

**Haim Brezis** (Rutgers U, Technion and Paris VI). *New approximations of the total variation and filters in Image Processing.*
Abstract. I will present new results concerning the approximation of the BV- norm by nonlocal, nonconvex, functionals. The mode of convergence introduces mysterious novelties and numerous problems remain open. The original motivation comes from Image Processing.

**Jong-Shi Pang** (University of Southern California). *Differential Variational Systems: A New Paradigm of Constrained Dynamical Systems.*
Abstract. This talk presents a new paradigm of constrained dynamical systems that we call a differential variational system. Such a system comprises an ordinary differential equation parameterized by an algebraic variable that is constrained to be a solution of a finite-dimensional state-dependent variational inequality. A differential variational system provides a very broad, unifying modeling framework for many special cases, including a differential algebraic equation, an optimal control problem with state-control constraints, a constrained differential Nash game, a hybrid switch system of a certain kind, as well as all biological, systems undergoing time evolution and subject to activation or de-activation of constraints. Many challenging mathematical issues arise as a result of the differential and variational features of such a system, requiring new treatment combining both classical theory of ordinary differential equations and results of contemporary optimization and variational inequalities. A summary of such issues will be presented and a touch of some selected results will be discussed.

**Boris Mordukhovich** (Wayne State University). *Optimal Control of the Sweeping Process over Moving Sets.*
Abstract. We study a new class of optimal control problems of the sweeping (Moreau) process governed by differential inclusions described by the

normal cone mapping to a controlled moving set in finite dimensions. The main attention is paid to developing the method of discrete approximations, justifying its well-posedness, and using it to derive necessary optimality conditions for such unbounded and discontinuous differential inclusions with state constraints. This is done by employing appropriate tools of second-order variational analysis and generalized differentiation.

**Dmitriy Drusvyatskiy** (University of Waterloo). *Feasibility Problems: from Alternating Projections to Matrix Completions.*

Abstract. Feasibility problems permeate computational mathematics. In the most general framework, we may seek a point in the intersection of two arbitrary closed sets. I will revisit an intuitive and widely used algorithm for such problems due to Van Neumann - the method of alternating projections. The algorithm proceeds by projecting an iterate onto one set, then onto the next, and so on. For two sets intersecting transversely, the method converges linearly. Reassuringly, for semi-algebraic intersections — prototypical pathology free feasibility problems — "generic" intersections are indeed transversal. On the other hand, for highly structured problems (such as those coming from graph embeddings), the two relevant sets tend to intersect only "tangentially". I will illustrate, by focusing on the Euclidean distance completion problem, how this seemingly complicating feature can instead be used as an advantage. Joint work with V. Cheung (Waterloo), A.D. Ioffe (Technion), N. Krislock (NIU), A.S. Lewis (Cornell), G. Pataki (UNC), H. Wolkowicz (Waterloo).

**Alexander Ioffe** (Technion). *Variational Analysis and Necessary Optimality Conditions.*

Abstract. In the talk we shall briefly survey some basic ideas and techniques variational analysis may add to the study of necessary optimality conditions, mainly with regard to optimal control problems. In particular, I shall describe a well structured and much shorter proof of a slightly improved version of the, so far the most advanced, maximum principle for systems governed by differential inclusions proved by Clarke a decade ago.

**Aharon Ben-Tal** (Technion). *Tractable Solutions to Some Challenging Optimization Problems.*

Abstract. The need to solve real-life optimization problems poses frequently a severe challenge, as the underlying mathematical programs threaten to be intractable. The intractability can be attributed to any of the following properties: large dimensionality of the design dimension; lack of convexity; parameters affected by uncertainty. In problems of designing optimal mechanical structures (truss topology design, shape design, free material optimization), the mathematical programs typically have hundreds of thousands of variables, a fact which rules out the use of advanced modern solution

methods, such as Interior Point. The same situation occurs in Medical Imaging (reconstruction of clinically acceptable images from Positron Emission Tomographs). Some Signal Processing and Estimation problems may result in nonconvex formulations. In the wide area of optimization under uncertainty, some classical approaches, such as chance (probabilistic) constraints, give rise to nonconvex NP-hard problems. Nonconvexity also occurs in some Robust Control problems. In all the above applications we explain how the difficulties were resolved. In some cases this was achieved by mathematical analysis, which converted the problems (or its dual) to a tractable convex program. In other cases novel approximation schemes for probability inequalities were used. In the case of huge-scale convex programs, novel algorithms were employed. In the Robust Control example, a reparameterization scheme is developed under which the problem is converted to a tractable deterministic convex program.

**Marc Teboulle** (Tel-Aviv University). *Proximal Alternating Linearized Minimization for Nonconvex and Nonsmooth Problems.*
Abstract. We introduce a proximal based algorithm for solving a broad class of nonconvex and nonsmooth minimization problems. It relies on a blend of first order methods which exploits data information and structure. We outline a self contained convergence analysis framework describing the main tools and methodology. We prove that the sequence generated by the proposed scheme globally converge to a critical point. Our approach allows to analyze various classes of nonconvex nonsmooth problems and related algorithms with semi algebraic problem's data, the later property being shared by a wealth of problems arising in a wide variety of applications. As an illustration of our results, we derive a new an simple globally convergent algorithm for solving nonnegative matrix factorization problems with sparsity constraints. Joint work with Jerome Bolte (Toulouse) and Shoham Sabach (Gottingen).

**Itai Shafrir** (Technion). *Asymptotics of Eigenstates of Elliptic Problems with Mixed Boundary Data Set on Domains Tending to Infinity.*
Abstract. We analyze the asymptotic behavior of eigenvalues and eigenfunctions of an elliptic operator with mixed boundary conditions on cylindrical domains when the length of the cylinder goes to infinity. We identify the correct limiting problem and show in particular, that in general the limiting behavior is very different from the one for the Dirichlet boundary conditions. Joint work with Michel Chipot and Prosenjit Roy from the University of Zurich.

## 7. Qualitative and Analytic Theory of ODE's

**F. Pakovich** (Ben-Gurion University). *Mixed moment problem and parametric center problem for Abel differential equation.*

Abstract. We will present recent results concerning polynomial solutions of the system

$$\int_a^b P^i dQ = 0, \qquad \int_a^b Q^i dP = 0, \quad i \geq 0,$$

and applications of these results to the parametric center problem for the Abel differential equation.

**D. Novikov** (Weizmann Institute). *Multiplicity Operators and Effective bounds on multiplicities of non-isolated intersections of Noetherian functions.*

Abstract. A subring of the ring of functions of several complex variables holomorphic in some domain is called a ring of Noetherian functions if it is closed under differentiation, contains polynomials and is Noetherian. There is a natural conjecture that the local complexity of objects defined by Noetherian functions can be bounded from above by their natural complexity parameters, such as degrees of polynomials used in their definition and dimensions. The simplest case of this conjecture is an upper bound on the multiplicity of an isolated common zero of several Noetherian functions proved by Gabrielov and Khovanskii in 1998. I will discuss a generalization of this bound to non-isolated common zeros, which, due to Morse theory, should apparently imply an answer to the general conjecture.

The main tool is the theory of multiplicity operators: these are partial differential operators extending algebraic local properties of zeros of holomorphic mappings to metric properties of their zero sets in some small disk around zero.

Joint work with G. Binyamini.

**Antonio Lerario** (University of Lyon). *Topology of intersections of random quadrics.*

Abstract. Betti numbers $b_{2i}(X_{\mathbb{C}})$ of a complete intersection $X_{\mathbb{C}}$ in complex projective space distributes in the range $0, \ldots, \dim(X_{\mathbb{C}})$ in a "delta shaped" function: they are all "ones" except in the middle dimension where all the topology is concentrated (the sum of all Betti numbers is represented by the "integral" of this function).

Looking at the real analogue of this picture the only thing we can say is that the sum ("integral") of all the Betti numbers of the real part $X_{\mathbb{R}}$ is bounded by that of its complex counterpart $X_{\mathbb{C}}$.

Despite this high level of freedom over the reals, we can still ask what is the typical behavior of this picture. In other words, picking a random real complete intersection, what do we expect its Betti numbers to be? And how do we expect them to distribute in the range $0, \ldots, \dim(X_{\mathbb{R}})$?

In this talk I will combine techniques from algebraic topology (spectral sequences) and probability theory (random matrices) to give an answer to these questions in the case of an intersection of random quadrics. I will show that, in a sense, the real picture is on average the "square root" of the complex one. Joint work with E. Lundbe.

**G. Binyamini** (University of Toronto). *Multiplicity estimates for solutions of ordinary differential equations.*
Abstract. The problem of estimating the multiplicity of the zero of a polynomial when restricted to the trajectory of a non-singular polynomial vector field, at one or several points, has been considered by authors in several different fields. The two best (incomparable) estimates are due to Gabrielov and Nesterenko.
We present a refinement of Gabrielov's method which simultaneously improves these two estimates. Moreover, we give a geometric description of the multiplicity function in terms of certain naturally associated *polar varieties*, giving a topological explanation for an asymptotic phenomenon that was previously obtained by elimination theoretic methods in the works of Brownawell, Masser and Nesterenko.

**M. Zhitomirskii,** (Technion). *Completely Symmetric Centers.*
Abstract. A phase portrait of a vector field on a plane is called completely symmetric if it is invariant with respect to the group $\{i_1, i_2, i_1i_2, id\}$, where $i_1$ and $i_2$ are involutions. The simplest example is a local center defined by the germ of an analytic vector field with a non-degenerate linear approximation. The talk is devoted to the classification of completely symmetric centers defined by germs of vector fields with a nilpotent linear approximation and by germs of vector fields with zero 2-jet and generic 3-jet.

**Vladimir Rovenski** (Haifa University). *The mixed scalar curvature flow on a foliated manifold.*
Abstract. The mixed scalar curvature $S_{mix}$ of a foliated manifold $(M^{n+p}, F^p, g)$ is the averaged mixed sectional curvature. We ask: When a foliation admits a metric of positive/negative or constant $S_{mix}$? (see V.Rovenski and L.Zelenko: Prescribing the positive mixed scalar curvature of totally geodesic foliations, in Proc. "Foliations-2012", World Sci., 2013, 163 - 203). We examine the question using the flow of metrics $dg/dt = -2(S_{mix}(g) - Phi)g^{\perp}$, where $g^{\perp}(X,Y) = g(X^{\perp}, Y^{\perp})$ and $\Phi : M \to \mathbb{R}$ is leaf-wise constant. The flow preserves harmonic foliations. If the mean curvature of the normal distribution $D$ obeys $H = -n\nabla^F(\log u)$, then $du/dt = n\Delta u + (nbeta + Phi)u + Psi1u^{-1} - Psi2u^{-3}$. Here $\Delta$ is the leaf-wise Laplacian, $\beta(x)$ is a nonnegative non-umbilicity measure of $D$, and nonnegative functions $\Psi_1(x)$ and $\Psi_2(x)$ represent the norms of the second fundamental tensor of F and the integrability tensor of $D$, respectively. We study asymptotic behavior of

solutions and attractor of above equation using spectral parameters of leafwise Schrodinger operator $-\Delta - \beta$. We compare the solution with solution of ODE with constant coefficients: $du/dt = -\lambda u + \Psi_1 u^{-1} - \Psi_2 u^{-3}$, and apply the maximum principle. As a result, we find sufficient conditions when there exist D-conformal to $g$ metrics on $(M, F)$ with $S_{mix}$ positive, negative or constant. For warped products, the PDE reads as the heat equation $du/dt = n\Delta u$, where u is the warping function, and we illustrate solutions for particular case: surfaces of revolution foliated by meridian curve.

**Vladimir Shikhman** (Center for Operations Research and Econometrics (CORE), Catholic University of Louvain (UCL)). *Algorithmic models of market equilibrium.*
Abstract. In this talk we suggest a new framework for constructing mathematical models of market activity. Contrary to the majority of the classical economical models (e.g. Arrow-Debreu, Walras, etc.), we get a characterization of general equilibrium of the market as a saddle point in a convex-concave game. This feature significantly simplifies the proof of existence theorems and construction of the adjustment processes both for producers and consumers. Moreover, we argue that the unique equilibrium prices can be characterized as a unique limiting point of some simple price dynamics. In our model, the equilibrium prices have natural explanation: they minimize the total excessive revenue of the market's participants. Due to convexity, all our adjustment processes have unambiguous behavioral and algorithmic interpretation. Joint work with Yu. Nesterov.

**Dmitry Batenkov** (Ben-Gurion University). *On the polynomial moment vanishing problem.*
Abstract. We consider the following problem: given two polynomials $p, q$ and a complex curve $\gamma$, give a "uniform" upper bound on the number $M$ for which the vanishing of the first $M$ moments $m_k(p, q, \gamma) = \int_\gamma p^k q$ implies the vanishing of the rest of the moments. This problem is deeply connected to the Center-Focus and Smale-Pugh problems for the Abel differential equation. Our approach is based on considering $m_k$ as usual moments of a certain algebraic function $g$, and analyzing a linear differential equation satisfied by the Cauchy transform of $g$. Joint work with Gal Binyami.

**Alex Brudnyi** (University of Calgary). *On Characterization of Universal Centers of ODEs with Analytic Coefficients.*
Abstract. We present a solution of the problem of characterization of universal centers of a differential equation $v' = \sum_{j=1}^{n} a_j v^{j+1}$ with all $a_j$ real analytic in a neighbourhood of $[a, b] \Subset \mathbb{R}$ in terms of vanishing of finitely many moments determined by $a_1, \ldots, a_n$.

## 8. Quasigroups, Loops and application

**Luise-Charlotte Kappe** (Binghamton University). *Finite coverings: a journey through groups, loops, rings and semigroups.*

Abstract. A group is said to be covered by a collection of subsets if each element of the group belongs to at least one subset in the collection: the collection of subsets is called a covering of the group.
On the bottom of page 105 of Derek Robinson's "Finiteness Conditions and Generalized Soluble Groups I", there are two theorems which served as my roadmap for exploring finite coverings of groups, loops, rings and semigroups. The first one is an unpublished result by Reinhold Baer: Baer's Theorem: A group is central-by finite if and only if it has a finite covering by abelian subgroups.
The second one, due to Bernhard Neumann, is stated as follows: Neumann's Lemma: Let G be a group having a covering by finitely many cosets by not necessarily distinct subgroups. If we omit any cosets of subgroups of infinite index, the remaining cosets will still cover the group.
In my talk I will report on my journeys through groups, loops, rings and semigroups, on what I discovered there about finite coverings together with several fellow travelers and on some discoveries which might still lie ahead.

**Risto Atasanov** (Western Carolina University). *Loops that are partitioned by groups.*

Abstract. A set of subgroups of a group is said to be a partition if every nonidentity element belongs to one and only one subgroup in this set. In this talk we will discuss structure of loops that are partitioned by subgroups.

**Tuval Foguel** (Western Carolina University). *Subloop Lattices.*

Abstract. We will discuss some topics from the theory of subgroup lattices. After giving a general overview, we will look at major open problem: if every finite lattice occurs as an interval in the subgroup lattice of a finite group. Next we investigate lattice of subloops of finite loops that are not lattices of subgroups of a finite group.

**Stephen Gagola III** (University of Witwatersrand). *The existence of a nontrivial nucleus.*

Abstract. An open problem, originally proposed by J.D. Phillips, asks if there exists an odd ordered Moufang loop that possesses a trivial nucleus. In this presentation, we will observe under what assumptions will a finite Moufang loop be guaranteed a nontrivial nucleus. We will also discover precisely what part of such a loop will have to fall inside of the nucleus.

**Jonathan Smith** (Iowa State University). *Directional quasigroups and their quasigroup replicas.*

Abstract. Directional quasigroups, or diquasigroups, are sets carrying both

a left and a right quasigroup structure. As such, they avoid the symmetry-breaking inherent in the selection of a left or right quasigroup alone. We will examine several new instances of diquasigroups, including dicores and group-representable diquasigroups.
Attention then turns to undirected replicas of diquasigroups, which are the largest quotients where the left and right quasigroup structures coincide. As two-sided quasigroups, these replicas offer a new approach to the construction of quasigroups of various kinds. Most generally, free quasigroups are recognized as the undirected replicas of free diquasigroups. Joint work with K. Matczak.

**Maria de Lourdes Merlini Giulian** (Universidade Federal do ABC). *Some Automorphic Loops and their automorphism group.*
Abstract. Automorphic loops or A-loops are loops in which all inner mapping are automorphisms. I will talk about two types of A-loops, one being a generalization of the dihedral group and the other one is obtained from a Lie ring. The goal is to define and identify the group of automorphisms of these two types of loops. Also, I will present their inner mapping groups.

**Izabella Stuhl** (University Debrecen). *Steiner Loops.*
Abstract. Steiner triple systems play a major part in combinatorics; many interesting connections have been developed between their combinatorial and algebraic aspects. From this point of view, the study of their algebraic background can be useful.
This generates an interest towards Steiner quasigroups and loops. In this presentation we analyse their multiplication and automorphism groups. Specifically, we discuss which groups can be multiplication groups of Steiner loops (this concept is important for non-associative structures). This question has been solved for several classes of Steiner quasigroups and loops. Furthermore, we prove that all automorphisms of a free Steiner loop are tame, and the automorphism group cannot be finitely generated when the loop has more than 3 generators.
The automorphism group of the 3-generated free Steiner loop is generated by the symmetric group $S_3$ and by the elementary automorphism $\varphi = e_1(x2)$. We also conjecture that $\mathrm{Aut}(S(x1, x2, x3))$ is the Coxeter group $\langle (12), (13), \varphi | (\varphi(12))^3 = (\varphi(13))^4 = ((12)(13))^3 = 1\rangle$. These conjecture fits the context of the work by U. Umirbaev on linear Nielsen-Schreier varieties of algebras.
We also introduce Steiner loops that are not Moufang yet satisfy Moufang's theorem and when do not satisfy Moufang's theorem.

**Ales Drapal** (Charles University). *The semidirect product in Moufang loops and its implications.*
Abstract. In the talk I will briefly restate the properties of the semidirect product construction of a Moufang loop and a cyclic group, as discovered by

Gagola, and will then show how this construction simplifies when the conditions upon the loops are strengthened. In particular it will be explained how centrality forces a behavior similar to that of alternating forms.

**Piroska Csorgo** (Eotvos Lorand University (ELTE)). *On Moufang loops of odd order.*
Abstract. Some sufficient (and neccessary) conditions for the existence of nontrivial nucleus, furthermore a sufficient and necessary condition for the existence of nontrivial center in Moufang loops of odd order.

**Peter Plaumann** (Universidad Autonoma "Benito Juarez" de Oaxaca). *The Cayley graph and the Growth of Steiner loops.*
Abstract. We use the variety of Steiner loops in order to check what one can expect from a *Combinatorial Theory of Loops* which should show analogies to the Combinatorial Group Theory. Doing this it turns out quite helpful that one can define a Steiner loop as a magma with a neutral element satisfying the identities $xy = yx, \quad (xy)y = x$. For a finite number $d \geq 0$ denote by $\S_d$ the free Steiner loop of rank $d$ and by $M_d$ the free magma with a unit element of rank $d$. Then we have a natural epimorphism $\eta_d : M_d \to \S_d$. Counting the number of words in $M_d$ is very easy. From the result one immediately obtains the
**Theorem 1.** For $d \geq 3$ the free Steiner loop $\S_d$ has exponential growth.
For a Steiner loop $S$ and a subset $E \subseteq S$ one defines the *Cayley graph* $\mathrm{Cay}_E(S)$ calling elements $x, y \in S$ adjacent if there is an element $u \in E$ such that $y = ux$.
**Theorem 2.** For $d \geq 3$ the Cayley graph of the free Steiner loop $\S_d$ with respect to a basis is not connected.
The Theorem 2 indicates difficulties to generalize the machinery of the Combinatorial Group Theory to the case of loops even for Steiner loops. Another handicap is the fact that a subloop of finite index of $\S_d$ with $d$ finite is not necessarily finitely generated.For the following question we not know an answer.
**Question 1.** Are there finitely generated Steiner loops of intermediate growth? Joint work with L. Sabinina and I. Stuhl.

**Liudmila Sabinina** (Universidad Autonoma del Estado de Morelos). *Coverings of automorphic Moufang loops.*
Abstract. For a Moufang loop $M$ we use the notation $D(M) = [M, M] \cap (M, M, M)$. We define a *covering* of a loop $Q$ as a pair $(Q_1, \eta)$, such that $\eta : Q_1 \to Q$ is an epimorphism. There is a natural partial order on the set of all coverings of a given loop $Q$.
For an automorphic Moufang loop $A$ let $\mathcal{P}(A)$ be the set of all coverings $(X, \eta)$ of $A$ for which $D(X) = 1$.
We will show that for any automorphic Moufang loop $A$ there exist minimal elements $(A_1, \eta)$ in $\mathcal{P}(A)$. We will study these minimal elements and will

show that en general they are not uniquely determined. Joint work with A. Grishkov and M. Rasskazova.

**Jonathan Hall** (Michigan State University). *Axial algebras of Jordan type.*
Abstract. An axial algebra over the field $F$ is a commutative algebra generated by idempotents whose adjoint action have multiplicity-free minimal polynomial. For semisimple associative algebras this leads to sums of copies of $F$. Here we consider the first nonassociative case, where adjoint minimal polynomials divide $(x-1)x(x-\eta)$ for fixed $0 \neq \eta \neq 1$. Jordan algebras arise when $\eta = 1/2$, but our motivating examples are certain Griess algebras of vertex operator algebras and the related Majorana algebras. We study a class of algebras, including these, for which axial automorphisms like those defined by Miyamoto exist, and there classify the 2-generated examples. For $\eta \neq 1/2$ this implies that the Miyamoto involutions are 3-transpositions, leading to a classification. Joint work with Felix Rehren and Sergey Shpectorov.

**J.D. Phillips** (Northern Michigan University). *On finite Bruck loops.*
Abstract. We present some recent results on (left) Bruck loops, emphasizing the enigmatic behavior of the right nucleus.

**Markku Niemenmaa** (University of Oulu). *Some solvability criteria for finite loops.*
Abstract. In 1996 Vesanen managed to show the following: if $Q$ is a finite loop and the multiplication group $M(Q)$ is a solvable group, then $Q$ is a solvable loop. In my talk I will consider those properties of the inner mapping group $I(Q)$ which guarantee the solvability of $M(Q)$ and thus the solvability of the loop $Q$.

**Petr Vojtechovsky** (University of Denver). *Abelian extensions and solvable loops.*
Abstract. Based on the recent development of commutator theory for loops, we provide both syntactic and semantic characterization of abelian normal subloops. We highlight the analogies between well known central extensions and central nilpotence on one hand, and abelian extensions and congruence solvability on the other hand. In particular, we show that a loop is congruence solvable (that is, an iterated abelian extension of commutative groups) if and only if it is not Boolean complete, reaffirming the connection between computational complexity and solvability. Joint work with David Stanovský.

**Mikhail Klin** (Ben-Gurion University). *Prolific construction of directed strongly regular graphs with the aid of loops.*
Abstract. A concept of directed strongly regular graph (briefly DSRG) is a generalization of classical strongly regular graphs to the case of directed

graphs. DSRGs were introduced in 1988 by Art Duval together with a number of feasible conditions on their parameter sets and first classes of examples. We start from the presentation of examples of DSRGs with new parameter sets $(50, 18, 6, 7, 12)$, $(72, 22, 6, 8, 14)$, $(98, 26, 6, 9, 16)$. Analysis of the symmetry properties of the discovered (with the aid of a computer) new graphs allows to conclude that, in fact, we are approaching a new infinite family of DSRGs with $2n^2$ vertices, $n > 1$, which correspond in an unified manner to an arbitrary loop of order $n$. Thus we obtain a new family of DSRGs, which is prolific, in the sense that the number of non-isomorphic DSRGs grows exponentially with $n$.
This research is in part supported by the Mobility Project at Matej Bel University, Slovakia, ITMS code: 26110230082. Joint work with Stefan Gyurki of Slovak University of Technology.

## 9. Algebraic Groups, Division Algebras and Galois Cohomology

**Gopal Prasad** (University of Michigan). *Weakly commensurable groups and locally symmetric spaces.*
Abstract. I will discuss the notion of weakly commensurable Zariski-dense subgroups of semi-simple algebraic groups. This notion was introduced in my joint work with Andrei Rapinchuk (Publ. math. IHES 109 (2009), 113-184), where we determined when two Zariski-dense $S$-arithmetic subgroups of absolutely almost simple algebraic groups over a field of characteristic zero are weakly commensurable. These results enabled us to prove that in many situations isospectral locally symmetric spaces of simple real algebraic groups are necessarily commensurable.

**Alena Pirutka** (Ecole Polytechnique, France). *Classifying spaces and counterexamples to the integral Tate conjecture.*
Abstract. In this talk we will discuss a counterexample to an integral version of the Tate conjecture. Following the approach of Atiyah-Hirzebruch and Totaro, we will use Steenrod operations and classifying spaces of algebraic groups to construct a non-torsion non-algebraic cohomology class. Joint work with N. Yagita.

**Yoav Segev** (Ben-Gurion University). *On graphs and valuations.*
Abstract. In this talk I will briefly outline techniques to construct valuations on a division ring $D$, both commutative and non-commutative, given a normal subgroup $N$ of $D^*$ of finite index. These techniques are based on a certain "valuation graph" on the vertex set $D^*/N$, the prototype of which are the Milnor K-graph and the commuting graph respectively. This unifies and extends earlier results. Joint work with Ido Efrat and Andrei

Rapinchuk, see `arxiv.org/abs/1401.8213` for more deta.

**Eva Bayer-Fluckiger** (EPFL, Switzerland). *Embeddings of maximal tori in classical groups and algebras with involution.*
Abstract. Embeddings of maximal tori into classical groups over global fields of characteristic $\neq 2$ are the subject matter of several recent papers (for instance by Prasad and Rapinchuk, Fiori, Lee), with special attention to the Hasse principle. The aim of this talk is to describe a complete criterion for the Hasse principle to hold, and to give necessary and suffcient conditions for a classical group to contain a maximal torus of a given type. The embedding problem will be described in terms of embeddings of étale algebras with involution into central simple algebras with involution. Joint work with Parimala and Ting Yu Lee.

**Igor Rapinchuk** (Harvard University). *The genus of a division algebra.*
Abstract. Let $D$ be a finite-dimensional central division algebra over a field $K$. The genus $\mathbf{gen}(D)$ is defined to be the collection of Brauer classes $[D'] \in \mathrm{Br}(K)$, where $D'$ is a central division $K$-algebra having the same maximal subfields as $D$. I will discuss a finiteness result for the genus in the case that $K$ is a finitely generated field and also give some explicit estimations on the size of the genus in several situations.

**Danny Neftin** (University of Michigan). *Galois subfields of tame division algebras.*
Abstract. A central problem concerning division algebras is to determine which finite dimensional division algebras are crossed products, that is, admit a maximal subfield that is Galois over the center. We shall discuss a simple crossed product criterion for tamely ramified valued division algebras, and how it can be obtained using the theory of graded division algebras. Joint work with Timo Hanke and Adrian Wadsworth.

**Eli Matzri** (Ben-Gurion University). *Diophantine and cohomological dimensions.*
Abstract. We give explicit linear bounds on the $p$-cohomological dimension of a field in terms of its Diophantine dimension. In particular, we show that for a field of Diophantine dimension at most 4, the 3-cohomological dimension is less than or equal to the Diophantine dimension.

**Vladimir Chernousov** (University of Alberta, Canada). *Lower bounds for essential dimension of "adjoint" groups in characteristic 2.*
Abstract. Let $G$ be a simple adjoint group of rank $r$ defined over an algebraically closed field of characteristic 2. Let $c$ be an automorphism of $G$ of order 2 such that $c(t) = t^{-1}$ for all elements $t$ in a fixed maximal torus $T \subset G$. Denote by $\widetilde{G}$ the group generated by $G$ and $c$. Thus $\widetilde{G} = G$ for

types $A_1$, $B_r$, $C_r$, $D_r$ ($r$ even), $G_2$, $F_4$, $E_7$, $E_8$ and $(\widetilde{G} : G) = 2$ for types $A_r$ ($r \geq 2$), $D_r$ ($r$ odd), $E_6$. In the talk we will show that the essential dimension of $\widetilde{G}$ is at least $r + 1$. For fields of characteristic 0 this result was obtained by Reichstein and Youssin, and for fields of positive characteristic $p > 2$ this is due to Chernousov and Serre. Our result allows us to compute the precise value of the essential dimension of adjoint groups of type $B_r$, $C_r$ in characteristic 2 case and this shows that the lower bound $r+1$ of essential dimensions of simple groups can't be improved in general case. Joint work with A. Babic.

**David Harbater** (University of Pennsylvania). *Period-index bounds for competions of p-adic function fields.*
Abstract. Given a division algebra $A$ over a field $F$, there is an integer $e$ such that the index of $A$ divides the $e$-th power of its period. One can ask whether, for a given $F$, there is an integer $e$ that works for all $A$. We show that $e = 2$ for completions of function fields of $p$-adic curves, such as the fraction fields of $\mathbb{Z}_p[[x]]$ and of the $p$-adic completion of $\mathbb{Z}_p[x]$, as well as finite extensions of those fields. Joint work with Julia Hartmann and Daniel Krashen.

**Adam Chapman** (Universite catholique de Louvain). *Symplectic Groups, Quadratic Forms and Tensor Products of Quaternion Algebras.*
Abstract. In this talk we will discuss the connection between a certain generating set of the symplectic group and the chain equivalence for non-degenerate quadratic forms in characteristic 2. Then we shall discuss the connection between the latter and the chain equivalence for tensor products of quaternion algebras.

**Boris Kunyavski** (Bar-Ilan University). *Local-global properties of equations in simple matrix groups and algebras.*
Abstract. I will give a brief survey of local-global problems arising from equations of the form $w(x_1, \ldots, x_d) = a$ or $P(X_1, \ldots, X_d) = a$, where $w$ is a groupword on $d$ letters, $P$ is a noncommutative, associative (or Lie) polynomial, $a$ is a fixed element of a simple matrix group $G$ or a matrix algebra $A$, and solutions are sought among $d$-tuples of elements of $G$ (or $A$). Matrix entries are assumed to belong to a field (or ring) of arithmetical interest, such as a number field or its ring of integers.
The case of word equations in SL(2) (based on a joint work with T. Bandman) will be considered in more detail. In particular, there will be discussed a weak approximation theorem for varieties corresponding to commutator equations.

**Mikhail Borovoi** (Tel-Aviv University). *Stably Cayley semisimple groups.*
Abstract. Following Lemire, Popov and Reichstein, we call a linear algebraic group $G$ over a field $k$ a *Cayley group* if it admits a Cayley map, i.e., a $G$-equivariant birational isomorphism over $k$ between the group variety $G$ and its Lie algebra $\mathrm{Lie}(G)$. A prototypical example is the classical "Cayley transform" for the special orthogonal group $\mathrm{SO}_n$ defined by Arthur Cayley in 1846. A linear algebraic group $G$ is called *stably Cayley* if $G \times S$ is Cayley for some split $k$-torus $S$. We classify stably Cayley semisimple groups over an arbitrary field $k$ of characteristic 0. Joint work with Boris Kunyavskiĭ.

**Rony A. Bitan** (Bar-Ilan University). *A building-theoretic approach to relative Tamagawa numbers of quasi-split semisimple groups over global function fields.*
Abstract. Let $G$ be a semisimple quasi-split group defined over a global function field $K$. We express the (relative) Tamagawa number of $G$ in terms of local data including the number of types of special vertices in one orbit of the Bruhat-Tits building of $G(K_p)$ for some place $p$ and the class number of $G$ at $p$.

**Uriya First** (Hebrew University). *From systems of quadratic forms to single hermitian forms.*
Abstract. I will describe a general technique to extend various results about quadratic and hermitian forms to systems of non-unimodular sesquilinear forms. The technique is based on a certain categorical equivalence which allows one to reduce to the case of a single unimodular hermitian form. Results that can be extended in this manner include: Witt's cancellation theorem, Springer's theorem, the weak Hasse principle (in certain cases), and statements about finiteness of the genus.

**Stefan Gille** (University of Alberta, Canada). *On Brauer group of products of certain schemes with a form of a torus.*
Abstract. I will discuss criteria and give examples of schemes $X$ over a field of characteristic zero, such that the Brauer group of the product of $X$ with a form $Y$ of a torus $T$ is isomorphic to the Brauer group of $Y$. Particular examples of such $X$ are simply connected algebraic groups or affine quadrics. It turns out that in positive characteristic this "almost" never true.

**Eric Brussel** (California Polytechnic State University). *Computations in the Brauer group of $\mathbb{Q}_p(t)$.*
Abstract. We compute the homology of the degree-two Kato complex for the function field of a $p$-adic curve with respect to a regular 2-dimensional model over $\mathbb{Z}_p$, and also for the function field of the closed fiber.

## 10. Asymptotic Geometric Analysis

**Radoslaw Adamczak** (University of Warsaw). *The circular law for two models of non-Hermitian random matrices with dependent entries.*

Abstract. I will discuss two models of non-Hermitian random matrices with dependent entries for which the limiting spectral distribution turns out to be the circular law (as in the case of i.i.d. coefficients). The first model is given by an isotropic log-concave measure on the space of square matrices, unconditional with respect to the standard basis (joint work with D. Chafai).

The second model concerns matrices with exchangeable entries, satisfying certain normalization and growth conditions (joint work with D. Chafai and P. Wolff). In my talk I will focus on the aspects of the results related to asymptotic geometric analysis, i.e. obtaining lower bounds on the smallest singular value of the matrices in question and bounds on the distance of a single row of the matrix from the linear subspace spanned by some of the remaining rows.

**Mark Rudelson** (University of Michigan). *Small ball probabilities for linear images of high dimensional distributions.*

Abstract. We study concentration properties of random vectors of the form $AX$, where $X$ has independent coordinates and $A$ is a given matrix. We show that the distribution of $AX$ is well spread in space whenever the distributions of the coordinates of $X$ are well spread on the line. This extends the bound obtained by Paouris for log-concave measures to random vectors with independent coordinates. Joint work with Roman Vershynin.

**Ronen Eldan** (Microsoft Research, Redmond). *Applications of stochastic calculus to problems in Geometry and Analysis.*

Abstract. We will discuss several new results concerning high dimensional log-concave random vectors, Concentration of measure, Gaussian fields and Gaussian isoperimetry and noise stability whose proofs rely on new methods which are heavily based on the theory of stochastic calculus. By constructing a few new stochastic processes associated with our geometric objects, we will try to demonstrate how the latter can serve as a powerful new tool to prove theorems of geometric nature. In particular, we will present a recent joint work with James Lee in which we settle a conjecture posed by Talagrand about L-1 smoothing for the Ornstein-Uhlenbeck semigroup which can be seen as an extension of the Gaussian isoperimetric inequality.

**Apostolos Giannopoulos** (University of Athens). *Mean-width and mean-norm of isotropic convex bodies.*

Abstract. We discuss upper bounds for the mean-width $M^*(K) = \int_{S^{n-1}} h_K(x)\, d\sigma(x)$ and the mean-norm $M(K) = \int_{S^{n-1}} \|x\|\, d\sigma(x)$ of an isotropic symmetric convex body $K$ in $\mathbb{R}^n$; here, $h_K$ is the support function of $K$ and $\|\cdot\|$ is its

induced norm on $\mathbb{R}^n$. An essentially optimal upper bound for $M^*(K)$ was obtained recently by Emanuel Milman; he showed that

$$M^*(K) \leq C\sqrt{n}\log^2 n\, L_K,$$

where $L_K$ is the isotropic constant of $K$. In a joint work with E. Milman, we have shown that $M(K) \leq \frac{C\log^{2/5}(e+n)}{\sqrt[10]{n}L_K}$.
Both results follow from analogous estimates for the mean-width and the mean-norm of the $L_q$-centroid bodies of $K$. These, in turn, are based on a more general scheme; for example, we show that if $K$ is a centrally-symmetric convex body in $\mathbb{R}^n$ with $K \supseteq rB_2^n$ then $\sqrt{n}M(K) \leq C\sum_{k=1}^{n}\frac{1}{\sqrt{k}}\min\left(\frac{1}{r}, \frac{n}{k}\log\left(e+\frac{n}{k}\right)\frac{1}{v_k^{-}(}\right.$
where $v_k^{-}(K)$ denotes the minimal volume-radius of a $k$-dimensional orthogonal projection of $K$.

**Beatrice-Helen Vritsiou** (University of Paris VI). *Geometry of the $L_q$-centroid bodies of isotropic log-concave measures.*
Abstract. The $L_q$-centroid bodies of an isotropic log-concave measure have proven to encode plenty of useful information about the measure itself and its concentration properties. We present a few examples of geometric characteristics or parameters of the $L_q$-centroid bodies that can help in the study of questions such as the slicing problem.

**Elijah Liflyand** (Bar-Ilan University). *Fourier transform versus Hilbert transform.*
Abstract. We present several results in which the interplay between the Fourier transform and the Hilbert transform is of special form and importance.
1. In 50's, the following problem in Fourier Analysis attracted much attention: Let $\{a_k\}_{k=0}^{\infty}$ be the sequence of the Fourier coefficients of the absolutely convergent sine (cosine) Fourier series of a function $f : \mathbb{T} = [-\pi, \pi) \to \mathbb{C}$, that is $\sum |a_k| < \infty$. Under which conditions on $\{a_k\}$ the re-expansion of $f(t)$ ($f(t) - f(0)$, respectively) in the cosine (sine) Fourier series will also be absolutely convergent?
We solve a similar problem for functions on the whole axis and their Fourier transforms. Generally, the re-expansion of a function with integrable cosine (sine) Fourier transform in the sine (cosine) Fourier transform is integrable if and only if not only the initial Fourier transform is integrable but also the Hilbert transform of the initial Fourier transform is integrable.
2. The following result is due to Hardy and Littlewood: If a (periodic) function $f$ and its conjugate $\widetilde{f}$ are both of bounded variation, their Fourier series converge absolutely.
We generalize the Hardy-Littlewood theorem (joint work with U. Stadtmüller) to the Fourier transform of a function on the real axis and its modified Hilbert transform. The initial Hardy-Littlewood theorem is a partial case of this extension, when the function is taken to be with compact support.

3. These and other problems are integrated parts of harmonic analysis of functions of bounded variation. We have found the maximal space for the integrability of the Fourier transform of a function of bounded variation. Along with those known earlier, various interesting new spaces appear in this study. Their inter-relations lead, in particular, to improvements of Hardy's inequality.
There are multidimensional generalizations of these results.

**Artem Zvavitch** (Kent State University). *On Isomorphic version of the Busemann-Petty problem for the class of arbitrary measures.*
Abstract. The aim of this talk is to present properties of sections of convex bodies with respect to different types of measures. We will remind a formula connecting the Minkowski functional of a convex symmetric body $K$ with the measure of its sections. We apply this formula to study properties of general measures most of which were known before only in the case of the standard Lebesgue measure. In particular we will present a version of of the Isomorphic Busemann-Petty problem for arbitrary measures and its connections to the recient results of Alexander Koldobsky on the slicing inequality for aribitrary measures.

**Boaz Slomka** (Tel-Aviv University). *Functional covering numbers.*
Abstract. We will discuss the notions of classical and fractional covering numbers in the context of convex bodies, and focus on their extension for (mainly log-concave) functions. We will describe some of their properties as well as related inequalities. Joint work with Shiri Artstein-Avidan.

**Vitali Milman** (Tel Aviv University). *Extension of Minkowski's polarization result to quasi-concave functions and characterization of summations of convex sets.*
Abstract. In the first half of the talk, we discuss a functional extension of the classical Minkowki's theorem on polynomiality of volume. We construct a new, natural addition operation on the class of quasi-concave functions, and show that the (Lebesgue) integral is a polynomial with respect to this new sum. This allows us to define notions such as the "surface area" or the "mean width" of a quasi-concave function. We proceed to generalize some famous inequalities such as the Brunn-Minkowski inequality, the isoperimetric inequality and the Alexandrov inequalities.
In the second half of the talk, we turn our attention back to addition operations on convex sets. We show that under some mild assumptions, the Minkowski sum is the only addition under which the volume is a polynomial. We also give two theorems characterizing Lp-sums: one under an assumption about the induced homothety, and the other under some weak, general assumptions. ¡br¿ Based on joint works with Liran Rotem.

## 11. Recent trends in History and Philosophy of Mathematics

**Tiziana Bascelli** (Italy). *Geometric structures at the foundations of dynamics: Galileo's and Torricelli's science of motion.*
Abstract. Galileo presented his pioneering theory about local motion in the book Two New Sciences (1638), in the form of a geometric treatise. At the beginning of the book he presents the principle and definition from which, later on, he deduces properties of motion. In order to prove that a body falling from rest moves downward with a uniformly accelerated motion he has to introduce a new concept of speed and describe time, space, and speed by means of geometry. The application of geometry to objects that belong to natural philosophy implies that the main properties of both, geometric objects and those of the physical world are homogeneous. I will aim to show to which extent geometric objects match categories of natural philosophy. Galileo's principle of uniformly accelerated motion was not evident to his contemporaries, who questioned whether it was valid. Evangelista Torricelli supported Galileo's view but only after inverting the roles of the principle of accelerated motion and main theorems. He also developed further Galileo's scientific achievements applying principles of mechanics. I will aim to show Torricelli's proof of existence of a special acceleration for every solid body, the one we now call 'gravity'.

**Piotr Błaszczyk** (Pedagogical University of Cracow). *Continuum versus continuity. Zeno and his modern rivals revisited.*
Abstract. The thesis of the Achilles paradox is clear and precise: "in a race the quickest runner can never overtake the slowest" (Aristotle, *Physics*), "it is impossible for him [Achilles] to overtake the tortoise when pursuing it" (Simplicius, *On Aristotle's Physics 6*). However, in modern interpretations the question whether Achilles can overtake the tortoise is turned into the question whether he can catch up with it: "Achilles never catches the tortoise" (Black 1951), "Achilles will never catch up with [the tortoise]" (Vlastos 1967), "[Achilles] never catches the tortoise" (Huggett 2010). Moreover, the standard solution to the paradox consists in determining the "rendezvous point", where Achilles "comes abreast with the tortoise" (Grünbaum 1967). We show that there is a hidden assumption both in the very statement of the paradox and its standard solution that to overtake the tortoise Achilles has to catch up with it. We provide a mathematical model for the race such that Achilles overtakes the tortoise and there is no point at which he catches up with it. In our model the space is Euclidean, i.e. it represents the ancient Greek continuum, the race (movement) is continuous, and we can also represent in our model the standard solution to the Achilles paradox based on the modern notion of limit.

**Philip Ehrlich** (Ohio University Athens). *Cantorian and non-Cantorian Theories of Finite, Infinite and Infinitesimal Numbers and the Unification*

*Thereof.*
Abstract. In addition to Cantor's well-known systems of infinite cardinals and ordinals, there were a variety of other systems of actual infinite numbers that emerged in the decades bracketing the turn of the twentieth-century. Two grew out of the work of Paul du Bois-Reymond [1870-71, 1875, 1877, 1882], Otto Stolz [1883], Felix Hausdorff [1909] and G. H. Hardy [1910] on the rates of growth of real functions, and the others emerged from the pioneering investigations of non-Archimedean ordered algebraic and geometric systems by Giuseppe Veronese [1892], Tullio Levi-Civita [1892, 1898], David Hilbert [1899] and Hans Hahn [1907]. Unlike Cantor's systems, which solely embrace finite numbers alongside his familiar infinite numbers, the other just-said non-Cantorian number systems, like the more recent hyperreal number systems associated with Abraham Robinson's nonstandard approach to analysis, embody finite, infinite and infinitesimal numbers.
In [Ehrlich 2012], we show how the above-mentioned Cantorian and non-Cantorian number systems admit a striking unification in the author's [Ehrlich 2001] algebraico-tree-theoretic approach to J.H. Conway's system of surreal numbers. Building on the above, in this paper we will provide introductions to the aforementioned non-Cantorian theories of the finite, infinite and infinitesimal that emerged in the decades bracketing the turn of the twentieth-century, explain the motivation for their introduction, outline the roles these and related theories play in contemporary mathematics and discuss the relations between these theories and the better-known theories of Cantor and Robinson that emerge from the just-said unification. It is the author's hope that by drawing attention to the spectrum of theories of the infinite and the infinitesimal that have emerged from non-Archimedean mathematics since the latter decades of the 19th century, it will become clear that the standard 20th-century histories and philosophies of the actual infinite and the infinitesimal that are motivated largely by Cantor's theory of the infinite and by non-standard analysis are not only limited in scope but are inspired by an account of late 19th- and early 20th-century mathematics that is as mathematically myopic as it is historically flawed.

**Frederik Herzberg** (Universitat Bielefeld). *Epistemic justification, formally: Alternatives to foundationalism and an exercise in mathematical philosophy.*
Abstract. An important problem in epistemology concerns the question of finding necessary and/or sufficient conditions for beliefs to be justified (so-called "doxastic justification"). It has been commonly assumed, based on an influential interpretation of Aristotle's *Organon*, that non-skeptical positions with respect to doxastic justification fall within one of three categories: foundationalism, coherentism, and infinitism. Foundationalism claims that regresses of justification always have to end in some – foundational – belief; coherentism denies that the structure of doxastic justification is linear;

infinitism allows for infinite regresses of justification. For a long time, infinitism has not been considered a serious contender. In recent years, Peter Klein (1998ff) and Jeanne Peijnenburg (2007ff) have argued the case for infinitism from a traditional analytic and Bayesian epistemological perspective, respectively. Coherentism, however, has been in a defensive mood for more than two decades since impossibility theorems for coherence measures appeared and one of its chief proponents, Laurence BonJour, abandoned that position.
We shall show that a rigorous mathematical analysis entails: (i) that there is common ground between weak versions of foundationalism, coherentism, infinitism, (ii) that common arguments against infinitism fail on a large scale; (iii) that a graded, BonJourian notion of coherence admits a thoroughly prabilistic doxastic justification, using the Loeb measure construction known from nonstandard probability theory.
These findings possess plausibility for the epistemology of mathematics: Foundationalism is an account of full (as opposed to partial) justification of belief in mathematical propositions given an axiomatic foundation; coherentism accounts for partial justification (based, e.g., on competent authority); and infinite regress of justification seems an appropriate description of arguments regarding competing axiomatic foundations for mathematics.

**Emily R. Grosholz** (The Pennsylvania State University). *Ampliative Reasoning in Number Theory: Integers and the Complex Plane.*
Abstract. I argue, against logicizing philosophers of mathematics, that mathematical problem solving is typically ampliative, adding content that takes the problem solution beyond the original circumstances of the problem's articulation, often by reducing the original problem to a different problem in a different mathematical context. Such problem reduction cannot in general be encompassed by the strict theory reduction envisaged by philosophers who begin from predicate logic and set theory. The ingenious ways in which mathematicians embed problems in larger domains are various (their variety has never been adequately addressed by philosophers) and ampliative; just as important, the ways in which mathematicians re-insert problems back into the original context also present a variety worthy of philosophical scrutiny and discussion, and they are also ampliative. Mathematics grows sideways between domains in virtue of this incessant activity up-and-down the Jacob's ladder of mathematical research. So while the prudent advice of logicians who advise ontological parsimony can help to rein in mathematical speculation and to urge a due proportion between means and ends, I think it is also philosophically interesting to examine how the growth of mathematical knowledge in fact occurs. I take four different examples from the history of number theory to illustrate my point; in all four cases, number theorists use the complex plane as the larger context for their problem-solving in both analytic and algebraic number theory.
An extended version of the abstract is also available, `u.cs.biu.ac.il/ katzmik/grosholz14.pdf`.

**Eberhard Knobloch** (Institut fur Philosophie, Wissenschaftstheorie, Wissenschafts und Technikgeschichte, TU Berlin). *From Archimedes to Kepler: Analogies and the dignity of mathematics.*

Abstract. Archimedes used analogies in the context of discovery. Two examples will illustrate this method: the way in which he found the quantitative value of the surface of a sphere and the use of indivisibles in the strict sense of the word in order to calculate areas and volumes. Kepler admired Archimedes. He wrote his "New stereometry" explicitly referring to the ancient geometer. He liked analogies as his guide as he emphasized, especially when he calculated the volumes of solids. Without knowing Archimedes's "Approach" (falsely called "Method") he used a similar method applying not well defined infinitely small quantities. The most interesting and most difficult problem is the calculation of the volume of an ideal mathematical apple.

**Jean-Pierre Marquis** (University of Montreal). *The Evolution of the Idea of Pure Mathematics.*

Abstract. In this presentation, I will focus on the evolution of the notion of pure mathematics, especially in the 19th and 20th century. I will try to exhibit what I take to be various shifts in the notion itself, from the opposition between pure/mixed mathematics and pure/applied mathematics, to a general notion of pure mathematics based on the axiomatic and abstract method, which characterized prominently the 20th century. The latter notion lead to a view that the applications of mathematics to the real world was no less than a miracle. This latter claim could not have been made and was not even conceivable even at the beginning of the 20th century. I will conclude with remarks on the present situation suggesting that the notion is shifting again along two main axes. The first one has to do with endo-applications of mathematics to itself, via various mathematical bridges constructed in the last fifty years or so, thus bringing a dif̄erent unity to mathematics and a dif̄erent way of thinking of applications itself. The second axis is the development of categorical frameworks that bring at the same time a higher level of abstraction and a different way of thinking about applications.

**Patrick Reeder** (Kenyon College). *Locating success and failure in applying Nonstandard Analysis to Euler's Introductio.*

Abstract. In Leonhard Euler's seminal work *Introductio in Analysin Infinitorum* (1748), he readily used infinite numbers and infinitesimals in many of his proofs. We aim to reformulate a group of proofs from the *Introductio* using concepts and techniques from Abraham Robinson's celebrated Nonstandard Analysis (NSA). We will specifically examine Euler's proofs of the

Euler formula, the divergence of the harmonic series and—time permitting—the Wallis product. We argue that NSA possesses the tools to provide appropriate proxies of some–but not all–of the inferential moves found in the *introductio*. Each of these proofs relies upon familiar Taylor-MacLaurin series representations of functions like sine, cosine and the natural logarithm. We will show that once we assume the Taylor-MacLaurin series representation, the proofs glide straight through NSA. Now, Euler's own proofs for the Taylor-MacLaurin series also appear in the *introductio*. What is striking is that every single one breaks down within NSA. I hope to examine these breakdowns and then provide some (at least preliminary) diagnostic discussion as to why NSA trips over Euler's proofs of these historically significant infinite series.

**David Sherry** (Northern Arizona University). *Leibniz and the Syncategorematic Infinite.*
Abstract. Leibniz generally conceived of infinitesimals as fictions. I dispute the currently fashionable opinion that Leibniz's infinitesimals are best understood as logical fictions, fictions eliminable by paraphrase. This so-called syncategorematic conception of infinitesimals is present in Leibniz's texts, but there is an alternative, formalist account of infinitesimals there too. I argue that the formalist account makes better sense of Leibniz's frequent analogy between infinitesimals and imaginary roots. Moreover, the formalist interpretation fits better with Leibniz's deepest philosophical convictions.

**Mark Jay Steiner** (Hebrew Univesity). *The Silent Revolution in the Philosophy of Ludwig Wittgenstein, 1937.*
Abstract. It is customary to distinguish the early Wittgenstein of the Tractatus from the later Wittgenstein of the "Philosophical Investigations." But I believe a revolution of a similar magnitude occurred in the middle of the "later" period of Wittgenstein, August 1937, which included a new approach to the philosophy of mathematics. This approach is a change in the way Wittgenstein perceives the relationship between mathematics and its canonical applications. Instead of seeing mathematical theorems as rules of "grammar" governing mathematical language, he suddenly began to see them as rules governing human activities like counting and measuring. Mathematical theorems became "supervenient" upon these activities, which eliminated the "problem" of the applicability of mathematics.

## 12. History of Mathematics

**Jean Christianidis** (University of Athens). *The historiography of Diophantus, reconsidered.*
Abstract. Diophantus' *Arithmetica* has been characterized – since medieval

Islam, through the Renaissance and the Early Modern period and up to the leading historians of mathematics of the 20th century – as a book on algebra. This traditional approach has been criticized recently by some historians of mathematics who point out the anachronistic methodology that historians in the past were using in analyzing ancient texts. But criticizing the methodology by which one defends a historical claim does not mean necessarily that the claim itself is wrong. The talk discusses some crucial issues involved in Diophantus' problem-solving, thus, giving support to the traditional image about the character of Diophantus' work, but put in a totally new framework of ideas.

**Jeff Oaks** (University of Indianapolis). *Polynomials and equations in Arabic algebra.*
Abstract. The medieval Arabic concepts of polynomial and equation differ from ours. Unlike our polynomials, which are composed with the operations of addition/subtraction, scalar multiplication, and exponentiation, Arabic polynomials were regarded merely as aggregations of the powers of the unknown with no operations present. Arabic equations are (ideally) polynomial equations; thus they assert the equality of two such aggregations. The structure of the solutions to problems reflects this difference in that algebraists would work out all operations called for in the enunciation before they would set up an equation. The problems in Diophantus' *Arithmetica* and in medieval Italian algebra point to the same pre-modern concepts.

**Luis Puig** (University of Valencia). *Proofs and systems of signs in pre-symbolic algebra.*
Abstract. In pre-symbolic algebraic texts, one finds proofs of the algorithms for solving equations that rely on the use of geometrical figures in which the algebraic terms are represented. However, figures are not always used in the same way. We will compare the figures in the works of al-Khwarizmi, Thabit ibn Qurra, Umar al-Khayyam, and Sharaf al-Dınal-Tusı. However, proofs in pre-symbolic algebra do not always use geometrical figures. Already in al-Khwarizmı's book, one can find a proof of a calculation with algebraic expressions, in which he begins by saying "This does not admit any figure", and introduces a different kind of warrant: "As regards its necessity, it is clear by words" (*al-lafẓ*, expression). From al-Karajıonwards this kind of warrant developed into proofs that no longer use geometric figures, they are "algebraic".

**Veronica Gavagna** (University of Salerno). *Euclid in the Renaissance: the reworking of Frencesco Maurolico from an arithmetical viewpoint.*
Abstract. Francesco Maurolico (1494-1575) was one of the most interesting restorers of Greek mathematics during the Renaissance. His approach to the restoration of Classics was creative rather than philological, even in the case of Euclid's *Elements*. Among his extant writings we find a quite

faithful "reading" of some Books of the *Elements* (V, VII-X) , but the most innovative work is a compendium of the Euclidean text. Maurolico's *Elementorum Compendia* is not really a reasoned synthesis of the *Elements*, but rather a reworking influenced by a deep arithmetical interpretation of the topics, from the theory of proportions to the relationships between regular polyhedra. The compendium of the fifteen books of the *Elements* – grouped on the basis of the autograph instructions left by Maurolico – shows tracts of deep originality, but at the same time it constitutes a very heterogeneous work, raising several questions, both philological and mathematical.

**Tony Levy** (CNRS). *The Medieval Hebrew mathematics culture (XIIth-XVIth centuries): A general assessment.*
Abstract. I'll provide a definition of Hebrew mathematical culture, an overview of the period and persons of medieval Hebrew mathematics, and a survey of the bodies of mathematical knowledge available to medieval Jews, according to the current state of research. I'll underline some salient aspects of this specific mathematical tradition and suggest a definition of "Hebrew mathematics".

**Tzvi Langermann** (Bar-Ilan University). *Pythagoreanism in the Medieval Jewish Tradition.*
Abstract. Pythagoreanism is a concatenation of doctrines in the fields of arithmetic, astronomy, music, medicine, and ethics, bound together by a belief that "all things are number". Since antiquity, Jews have been attracted to Pythagoreanism, especially in the fields of ethics, astronomy, and arithmetic. My talk will focus on two aspects likely to be of interest to mathematicians: arithmology and its appearance in works of medieval Jewish philosophy, and a lengthy work by Qalonymos ben Qalonymos (1286-1329) that takes up in detail both the philosophical significance of numbers, as well as their arithmetic properties.

**Roy Wagner** (Hebrew University). *Citrabhanu's Twenty-One Algebraic Problems in Malayalam and Sanskrit.*
Abstract. This paper studies the Sanskrit and Malayalam versions of Citrabhanu's Twenty One problems – a discussion of quadratic and cubic problems from 16th century Kerala. It reviews the differences in the approaches of the two texts, highlighting the division between the Sanskrit integer arithmetic techniques and the Malayali fixed point reiterations. The paper concludes with some speculations on the possible transmission of algebraic knowledge between Kerala and the West.

**Karine Chemla** (CNRS and University of Paris Diderot). *Practices of abstraction in ancient China.*

Abstract. In 19th century historiography of mathematics, abstraction became a central value to describe – and assess – pieces of ancient mathematics. In this context, Greek writings such as Euclid's *Elements* or others, were prized for their abstraction, while writings produced in other parts of the world were downplayed on the same grounds, that is, for their apparent lack of abstraction, and their alleged exclusive emphasis on the achievement of practical goals. I have suggested that generality, more than abstraction, was a key value for practitioners of mathematics in ancient China. In this talk, I shall establish that abstraction was an actor's category in ancient China and discuss how it can be interpreted in this context. I shall then discuss the part played by abstraction in a world in which algorithms were an essential means of expression of mathematical knowledge. We shall see how abstraction relates to proof of the correctness of algorithms and examine how it is formulated.

**Ido Yavetz** (Tel-Aviv University). *Trigonometry, straight-edge and compass, and the applied mathematics of the Almagest.*
Abstract. Ptolemy's *Almagest* has no known Ancient rival in sheer mastery of applying computational techniques to astronomical data for the purpose of specifying the parameters of astronomical models. Completely self-contained, the work provides all the means required for the task – coordinate systems and transformations between them, the general physical principles that turn astronomy into a veritable *natural* mathematical science, and data to be used in the fully worked out examples. Of particular interest in this regard for the present paper is the systematic development of trigonometric means that are everywhere applied to the solution of each and every model. It turns out, however, that even the most complicated of these require no trigonometry at all, but may be solved as geometrical construction problems, using straight-edge and compass alone, with trigonometric iterations replaced by far more intuitive geometrical ones. It will then be shown that while the geometrical solution enjoys superior intuitive intelligibility, it cannot achieve the precision that Ptolemy obtains after three iterations, unless the diagrams can be accurately drawn with thin, precise lines on the scale of several meters. Thus, numerical computations seem preferable not only because they are easier to transmit and repeat than geometrical constructions, but also because they provide greater practical precision.

**Jesper Lützen** (University of Copenhagen). *Impossibility of the classical problems: algebraization equals mathematization.*
Abstract. My point here is that the impossibility question of the duplication of the cube, the trisection of the angle, and the quadrature of the circle remained a meta question until the algebraical methods opened up for a mathematical treatment of the impossibility question, first in the 17th century and finally in the 19th century.

**Tinne Kjeldsen** (Roskilde University). *Whose History? Minkowski's development of geometry of numbers and the concept of convex sets.*
Abstract. I plan to talk about methodological issues in history of mathematics with focus on interpretations, perspectives, and conceptions of history. I will use my research on the history of convexity as example.

**Michael Fried** (Ben-Gurion University). *Otto Toeplitz's "The problem of university infinitesimal calculus courses and their demarcation from infinitesimal calculus in high schools" (1927).*
Abstract. "The problem of university infinitesimal calculus courses and their demarcation from infinitesimal calculus in high schools" (1927) is the published version of an address Otto Toeplitz delivered at a meeting of the German Mathematical Society held in Düsseldorf in 1926. It contains the most detailed exposition of Toeplitz's ideas about mathematics education, particularly, his thinking about the role of the history of mathematics in mathematics education. The latter is what he called the "genetic method" to teaching mathematics. The tensions and assumptions about mathematics, history of mathematics and historiography revealed in this piece dedicated to educational ideas are what make Toeplitz's text interesting in the study of historiography of mathematics. In general, the ways historiography of mathematics and teaching of mathematics, even without an immediate concern for history, may be deeply entangled are, accordingly, worth attention both in historical and educational research.

## 13. Random Matrix Theory

**Sasha Sodin** (Princeton University). *Eigenvalue distribution of random matrices depending on an auxiliary variable, and a family of Airy-type random processes.*
Abstract. We discuss the eigenvalue distribution of a family of matrix-valued random processes, as the matrix size goes to infinity. The limiting object at the spectral edge is a random decreasing sequence of functions, which at every point has the distribution of an Airy point process.

**Gerard Ben Arous** (Courant Institute, New York University). *Smallest singular values for perturbations of random permutation matrices.*
Abstract. We take a first small step to extend the validity of Rudelson-Vershynin type estimates to some sparse random matrices, here random permutation matrices. We give lower (and upper) bounds on the smallest singular value of a large random matrix $D + M$ where $M$ is a random permutation matrix, sampled uniformly, and $D$ is diagonal. When $D$ is itself random with i.i.d terms on the diagonal, we obtain a Rudelson-Vershynin type estimate, using the classical theory of random walks with negative drift.

Joint work with K. Dang.

**Doron Lubinsky** (Georgia Tech). *Universality limits via orthogonal polynomial techniques.*
Abstract. We outline some results on universality limits for random Hermitian Matrices, that can be proved using orthogonal polynomial techniques. These include a variational principle for the m-point correlation function, and universality in measure for arbitrary compactly supported measures.

**Alan Edelman** (MIT). *The singular values of the GUE: surprises that we missed.*
Abstract. Some properties of the eigenvalues of Gaussian Unitary Ensemble (GUE) can instead be phrased in terms of the singular values of the same ensemble. By working in this restricted setting, essentially discarding the signs of the eigenvalues, we gain access to a surprising decomposition: the singular values of the GUE are distributed as the union of the singular values of two independent ensembles of Laguerre type. The structure of this decomposition reveals that several existing observations about large n limits of the GUE are in fact manifestations of phenomena that are already present for finite random matrices: in particular, we obtain a finite analog of the obvious geometric relationship between the Semi- Circle Law and Quarter-Circle Law describing level densities, and we also gain new insight into the log-normal behavior of the absolute value of the determinant of the nxn GUE for large n. The decomposition also provides an approach for computing the distribution of the smallest singular value of the GUE,which in turn permits the study of the leading order behavior of the condition number of matrices drawn from this ensemble. Joint work with M. LaCroix.

**Paul Bourgade** (Harvard University and IAS). *Local quantum unique ergodicity for random matrices.*
Abstract. For generalized Wigner matrices, I will explain a probabilistic version of quantum unique ergodicity at any scale, and gaussianity of the eigenvectors entries. The proof relies on analyzing the effect of the Dyson Brownian motion on eigenstates. Relaxation to equilibrium of the eigenvectors is related to a new multi-particle random walk in a random environment, the eigenvector moment flow. Joint work with H.-T. Yau.

**Alice Guionnet** (MIT). *Transport maps and universality.*
Abstract. In this talk we discuss the construction of approximate transport maps between matrix models and deduce universality of the local fluctuations of their spectrum. Joint work with F. Bekerman, A. Figalli.

**Boaz Nadler** (Weizmann Institute). *Roy's Test, Rank-One Alternatives and Random Matrix Theory.*
Abstract. Roy's largest root is an important statistical test appearing in a

variety of multivariate problems, including MANOVA, signal detection in noise, canonical correlation analysis and more. A key theoretical question is to understand its power against various alternatives. In this work, assuming multivariate Gaussian observations, we derive a simple yet accurate approximations for its distribution in certain settings of "concentrated noncentrality", in which the signal or difference between groups is concentrated in a single direction. Rather than using asymptotic random matrix theory results for infinitely large matrices, we instead appeal to a perturbation approach in the signal strength, which works well for small sized matrices, common in various applications. The results allow relatively simple power calculations for Roy's test and provide lower bounds on the minimal number of samples required to detect a given group difference, or a given signal strength. Joint work with Iain Johnstone (Stanford).

**Shahar Mendelson** (Technion). *The quadratic empirical process and the extremal singular values of random matrices with iid rows.*
Abstract. The quadratic empirical process indexed by a class of functions F, is defined by $f \to \frac{1}{N}\sum_{i=1}^{N} f^2(X_i) - Ef^2$, where $X_1, \ldots, X_N$ are selected independently according to an underlying measure. This process appears naturally in numerous problems in geometry, statistics, signal processes and other areas.
We will present several results on the supremum of this process, and in particular, focus on the way the bound is governed by the structure of the underlying class F. When applied to a class of linear functionals on $R^n$, indexed by the unit Euclidean sphere and relative to an isotropic measure on $R^n$, these results lead to an estimate on the largest and smallest singular values of the random matrix whose rows are $X_1, \ldots, X_N$.
We will show that in a rather general situation (e.g. when $X$ is supported in a Euclidean ball of radius that is proportional to $\sqrt{n}$ and the $L_p$ and $L_2$ norms of linear forms are equivalent for some $p > 8$), the singular values obey a quantitative version of the Bai-Yin asymptotics. Also, the smallest singular value can be bounded from below under very weak assumptions and without any constraint on the Euclidean norm of $X$.

**Roman Vershynin** (University of Michigan). *Delocalization of eigenvectors of random matrices with independent entries.*
Abstract. We develop a new, geometric, approach to delocalization for random matrices. This approach yields that an $n$ by $n$ random matrix $G$ with independent entries is completely delocalized. Suppose the entries of $G$ have zero means, variances uniformly bounded below, and a uniform tail decay of exponential type. Then with high probability all unit eigenvectors of $G$ have all coordinates of magnitude $O(n^{-1/2})$, modulo logarithmic corrections. Joint work with Mark Rudelson.

**Van Vu** (Yale University). *Random matrices: Law of the determinant.*
Abstract. In this talk, we start by a brief survey on this classical problem. Next, wefocus on current developments concerning limiting laws, in both symmetric and non-symmetric cases, and conclude with several open questions.

## 14. Field Arithmetic

**Yuri Zarhin** (Penn State U and Weizmann Inst.). *Finiteness theorems for Brauer groups of K3 surfaces over finitely generated fields.*
Abstract. Let $k$ be a field that is finitely generated over its prime subfield (e.g., a number field) and $\mathrm{Br}(k)$ is the Brauer group of $k$. Let $X$ be a $K3$ surface over $k$ and let $\mathrm{Br}(X)$ be the Brauer-Grothendick group of $X$, which is a commutative periodic group. The structure map $X \to \mathrm{Spec}(k)$ induces the group homomorphism $\mathrm{Br}(k) \to \mathrm{Br}(X)$, whose image is a subgroup of $\mathrm{Br}(X)$ that is denoted by $\mathrm{Br}_0(X)$. If $X$ has a $k$-point then $\mathrm{Br}_0(X)$ is canonically isomorphic to $\mathrm{Br}(k)$.
**Theorem 1** [1]. If $\mathrm{char}(k) = 0$ then the quotient $\mathrm{Br}(X)/\mathrm{Br}_0(X)$ is a finite group.
If $p$ is a prime and $A$ is a commutative periodic group then we write $A(\text{non-}p)$ for the subgroup of $A$ that consists of all elements, whose order is prime to $p$.
**Theorem 2** [3]. If $p := \mathrm{char}(k) > 2$ then $[\mathrm{Br}(X)/\mathrm{Br}_0(X)](\text{non-}p)$ is a finite group.
Next we deal with the case when $k$ is the field $\mathbb{Q}$ of rational numbers and a $K3$ surface $X$ is a special Kummer surface (attached to a product of elliptic curves $u^2 = f(x)$ and $v^2 = g(y)$) that is a smooth projective model of an affine surface $z^2 = f(x)g(y)$ where $f(x)$ and $g(y)$ are cubic polynomials over $\mathbb{Q}$ without multiple roots. We write $\mathbb{Z}$ for the ring of integers.
**Theorem 3** [2]. Suppose that $a = 5 + 35m$, $b = 7 + 35n$, $a' = 35m' + 1$, $b' = 35n' + 2$ where $m, n \in \mathbb{Z}$, $m$ is not congruent to 2 modulo 5, $n$ is not congruent to 4 modulo 7, and $m'$ and $n'$ are any integers.
If $X$ is the Kummer surface over $\mathbb{Q}$ with affine equation $z^2 = x(x-a)(x-b)y(y-a')(y-b')$ then $\mathrm{Br}(X) = \mathrm{Br}(\mathbb{Q})$.
References: [1] A.N. Skorobogatov, Yu. G. Zarhin, J. Algebraic Geometry 17 (2008), no. 3, 481-502; [2] A.N. Skorobogatov, Yu. G. Zarhin, J. reine angew. Math. 666 (2012), 115-140; [3] A.N. Skorobogatov, Yu. G. Zarhin, arXiv:1403.0849 [math.AG]. Joint work with Alexei Skorobogatov.

**Sebastian Petersen** (Unversität der Bundeswehr München). *Independence of l-adic Galois representations.*
Abstract. Let $K$ be a field and $L$ the set of all prime numbers. Let $X/K$ be a separated algebraic scheme and $i \in N$. We consider for every $L$

the representation $\rho_l : \mathrm{Gal}(K) \to Aut\mathbb{Q}_l(Hi(X\bar{K}, \mathbb{Q}))$ of the absolute Galois group $Gal(K)$ of $K$ on the étale cohomology group $H_i(XK, \mathbb{Q}_l)$. Let $\rho : \mathrm{Gal}(K) \to \prod_{l\in L} im(\rho_l)$ be the canonical map. Such a family $\{\rho_l\}_{l\in L}$ of representations is said to be almost independent if there exists a finite separable extension $E/K$ such that $\rho(Gal(E)) = \prod_{l\in L} \rho_l(\mathrm{Gal}(E))$. Serre proved in 2010: If $K$ is a number field, then the family $(\rho_l)_{l\in L}$ is almost independent. This is a useful piece of information when working with these representations. Moreover, the statement ties in well with a system of more general conjectures, e.g. with the so-called adelic openness conjecture. Two papers together with Böckle and Gajda and a paper of Cadoret and Tamagawa deal with variations and generalizations of the above theorem of Serre. We shall discuss these results along with some applications. Joint work with Gebhard Böckle and Wojciech Gajdaa.

**Ido Efrat** (Ben-Gurion University). *Filtrations of absolute Galois groups.*
Abstract. A widely open problem in modern Galois theory is to understand the group-theoretic structure of absolute Galois groups of general fields. We give new restrictions on this structure, related to filtrations of the group and to its unipotent representations. Joint work with Jan Minac.

**Michael Larsen** (Indiana State U). *Diophantine properties of fields with finitely generated Galois group.*
Abstract. I will discuss a number of related conjectures concerning the rational points of varieties (especially curves and abelian varieties) over fields with finitely generated Galois group and present some evidence from algebraic number theory, Diophantine geometry, and additive combinatorics in support of these conjectures.

**David Harbater** (Penn State U). *Quadratic forms in field arithmetic.*
Abstract. This talk concerns the arithmetic of quadratic forms over fraction fields $F$ of rings $R$ that arise from fields $k$ having the property $C_n$, or more generally the property $A_n(2)$, in the notation of Leep. In particular, we consider finite extensions $R$ of rings of the form $k[[x, t]]$ and $k[x][[t]]$, and obtain results about the $u$-invariants of the resulting fraction fields $F$. For example, $k$ may be taken to be the field $\mathbb{Q}_p$ or $\mathbb{Q}_p(z)$ or $\mathbb{Q}_p((z))$ for any prime number $p$. The proofs use patching methods. Joint work with Julia Hartmann and Daniel Krashen.

**Alena Pirutka** (Ecole Polytechnique, France). *Stable invariants and quartic threefolds.*
Abstract. In a context of the Lüroth problem, for $K$ a function field of a smooth projective variety $X$ over a field $k$, one can ask whether $K$ is a purely transcendental over $k$ (resp. a subfield of a purely transcendental extension of $k$, resp. becomes purely transcendental after adding some independent

variables), that is, if $X$ is rational (resp. unirational, resp. stably rational). In this talk we will discuss various invariants which allows to answer this questions for some classes of varieties, and more specifically, for quartic threefolds. By a celebrated result of Iskovskikh and Manin, no smooth quartic hypersurface in $\mathbb{P}^4_{\mathbb{C}}$ is rational. Using a specialisation method introduced by C. Voisin, as well as a method based on the universal properties of the Chow group of zero-cycles, we will show that a lot of such quartics are not stably rational. Joint work with J.-L. Colliot-Thélène.

**Andrew Obus** (University of Virginia). *Toward a Generalization of the Oort Conjecture.*
Abstract. The (local) Oort Conjecture (now a theorem of Obus-Wewers and Pop) states that any cyclic $G$-action on $k[[t]]$, where $k$ is algebraically closed of characteristic p, lifts to characteristic zero. If $G$ is not necessarily cyclic, but still has cyclic $p$-Sylow subgroup, lifting is no longer always possible, due to the so-called Bertini obstruction. We conjecture that the Bertin obstruction is the only obstruction to lifting in this case.

**Florian Pop** (University of Pennsylvania). *Evidence for pro-l birational phenomena.*
Abstract. Both the Grothendieck's birational anabelian geometry and Bogomolov's birational anabelian program are set up in such a way that eventually one has at hand -among other things- the birational geometry over an algebraically closed base field. There is though evidence that less is needed, namely that things could work in a pure pro-l setting (which thus does not provide the birational geometry over the algebraic closure of the base field). I plan to explain the terms and give evidence for my claims. This is work in progress.

**Danny Neftin** (University of Michigan). *The Sylow subgroups of the absolute Galois group of* $\mathbb{Q}$.
Abstract. Following Serre's question, the Sylow subgroups of the absolute Galois group of a $p$-adic field were studied and completely understood by Labute. However, the structure of the $p$-Sylow subgroups of the absolute Galois group of the field of rational numbers is much more subtle and mysterious. We make progress towards its determination via a surprisingly simple decomposition. Joint work with Lior Bary-Soroker and Moshe Jarden.

## 15. Additive Number Theory

**Alfred Geroldinger** (University of Graz, Austria). *Zero-Sum Sequences and their sets of lengths.*
Abstract. Let $G$ be an additive finite abelian group. A sequence over $G$ means a finite sequence of terms from $G$, which is unordered and repetition

of terms is allowed. We say that a sequence has sum zero if its terms add up to zero. The set $\mathcal{B}(G)$ of zero-sum sequences over $G$ forms a monoid, where the operation is simply the juxtaposition of sequences. The maximal length $\mathsf{D}(G)$ of a minimal zero-sum sequence is called the *Davenport constant* of $G$ (equivalently, $\mathsf{D}(G)$ is the smallest integer $\ell \in \mathbb{N}$ such that every sequence $S$ over $G$ of length $|S| \geq \ell$ has a zero-sum subsequence). Let $S$ be a zero-sum sequence. If $S = U_1 \cdot \ldots \cdot U_k$, where $U_1, \ldots, U_k$ are minimal zero-sum sequences, then $k$ is called the length of this factorization. The *set of lengths* $\mathsf{L}(S)$ is defined as the set of all possible factorization lengths $k \in \mathbb{N}$. Clearly, $\mathsf{L}(S) \subset \mathbb{N}$ is a finite nonempty subset of the positive integers. Moreover, $|\mathsf{L}(S)| = 1$ for all sequences $S$ over $G$ if and only if $|G| \leq 2$. If $|G| \geq 3$, then for each $N \in \mathbb{N}$ there is a zero-sum sequence $S_N$ over $G$ such that $|\mathsf{L}(S_N)| \geq N$. Sets of lengths of $\mathcal{B}(G)$ coincide with sets of lengths of any Krull monoid with class group $G$, and this connection motivates their study. We discuss various parameters controlling the structure of sets of lengths. These parameters and their study are closely related to the Davenport constant $\mathsf{D}(G)$ of the group.

**Julia Wolf** (University of Bristol, United Kingdom). *Towards a Sidorenko-type conjecture for sets.*
Abstract. A famous conjecture of Sidorenko states that for any bipartite graph H, the random graph G(n,p) has asymptotically the minimum number of copies of H, where the minimum is taken over all graphs on n vertices of edge density p. We explore an analogous problem for additive configurations contained in dense subsets of the cyclic group $Z_p$.

**Patrizia Longobardi** (University of Salerno, Italy). *Sums of dilates and small doubling problems in Baumslag-Solitar groups.*
Abstract. The Baumslag-Solitar groups are defined as follows:

$$BS(m,n) = \langle a, b \mid b^{-1}a^m b = a^n \rangle$$

where $m, n$ are integers. We concentrate on the groups $BS(1,n)$ and their subsets of the type

$$S = \{b^r a^{x_1}, b^r a^{x_2}, \ldots, b^r a^{x_k}\} = b^r a^A$$

where $r$ is a positive integer and $A = \{x_1, x_2, \ldots, x_k\}$ denotes a finite sequence of integer. In particular, $|S| = |A|$. First we notice that $S^2 = \{b^r a^{x_i} b^r a^{x_j} \mid x_i, x_j \in A\}$ satisfies the following equality:

$$S^2 = b^{2r} a^{n \star A + A}$$

where $n \star A + A = \{nx_i + x_j \mid x_i, x_j \in A\}$. Sets of type $n \star A + A$ are called sums of dilates. We apply known and new results concerning the size of sums of dilates in order to prove various results concerning the size of $S^2$ for $S = b^r a^A \subset BS(1,2)$. A new result on dilates is the following: If $A$ is a finite set of integers and $|A + 2 \star A| \leq 4|A| - 4$, then $A$ is a subset of an

arithmetic progression of size $\leq 2|A|-3$ . We also investigate the structure of arbitrary subsets of $BS(1,2)$ satisfying small doubling properties. We consider the submonoid

$$BS^{+}(1,2)=\{b^{m}a^{x}\in BS(1,2)\mid x,m\in Z,m\geq 0\}$$

of $BS(1,2)$. We prove that if $S$ is a finite non-abelian subset of $BS^{+}(1,2)$ and $|S^2|\leq \frac{7}{2}|S|-4$, then $S=ba^{A}$, where $A$ is a set of integers of size $|S|$, which is contained in an arithmetic progression of size less than $\frac{3}{2}|S|-2$ . Joint work with G.A. Freiman, M. Herzog, M. Maj and Y.V. Stanchescu.

**Mercede Maj** (University of Salerno, Italy). *Some results on small doubling in ordered groups.*
Abstract. Let $G$ be a group and suppose that a total order relation $\leq$ is defined on the set $G$. We say that $(G,\leq)$ is an *ordered group* if for all $a,b,x,y\in G$, the inequality $a\leq b$ implies that $xay\leq xby$. A finite subset $S$ of a group $G$ is said to satisfy the *small doubling property* if $|S^2|\leq \alpha|S|+\beta$, where $\alpha$ and $\beta$ denote real numbers, $\alpha>1$ and $S^2=\{s_1s_2\mid s_1,s_2\in S\}$. Our aim in this talk is to investigate the structure of finite subsets $S$ of *ordered groups* satisfying the small doubling property with $\alpha=3$ and small $\beta$'s, and also the structure of the subgroup generated by $S$. This is a step in a program to extend the classical Freiman's inverse theorems to nonabelian groups. Let $G$ be an ordered group and let $S$ be a finite subset of $G$ of size $|S|=k\geq 2$. We proved that if $|S|>2$ and $|S^2|\leq 3|S|-3$, then $\langle S\rangle$ is abelian, and if $|S^2|\leq 3|S|-4$, then $S$ is a subset of a geometric progression. In this talk we present some recent results concerning the structure of the subset $S$ of an ordered group and the structure of $\langle S\rangle$, if $|S^2|\leq 3|S|-3+b$, for some integer $b\geq 1$. We prove that if $|S|>3$ and $|S^2|\leq 3|S|-2$, then either $\langle S\rangle$ is abelian and at most 3-generated, or $\langle S\rangle$ is 2-generated and one of the following holds: (i) $\langle S\rangle=\langle a,b\mid [a,b]=c,[c,a]=[c,b]=1\rangle$, (ii) $\langle S\rangle$ is the Baumslag-Solitar group $B(1,2)$, i.e. $\langle S\rangle=\langle a,b\mid a^{b}=a^{2}\rangle$; (iii) $\langle S\rangle=\langle a,b\mid [a,b]=c,[c,a]=1,(c)^{b}=c^{2}\rangle$; (iv)$\langle S\rangle=\langle a,b\mid ba^{2}=ab^{2},a^{2}ba^{-2}=bab^{-1}\rangle$. In particular, $\langle S\rangle$ is metabelian, and if it is nilpotent, then its nilpotence class is at most 2. Moreover, if $\langle S\rangle$ is abelian and $|S^2|\leq 3k-2$, then the set $S$ has Freiman dimension at most 3, and the precise structure of $S$ follows from some previous results of G. A. Freiman. More generally, if $\langle S\rangle$ is abelian and $|S^2|\leq (c+1)|S|-c(c+1)/2$, then the Freiman dimension of $S$ is at most $c-1$ and the group $\langle S\rangle$ is at most $c$-generated. Joint work with G. A. Freiman, M. Herzog, M. Maj, A. Plagne and Y. V. Stanchescu.

**Sergei Konyagin** (Moscow Lomonosov State University, Russia). *Beurling-Helson theorem and Fourier transform of characteristic functions of subsets of $\mathbb{Z}_p$.*
Abstract. Let $A(\mathbb{T})$ be the space of complex continuous functions $f$ on the unit circle $\mathbb{T}=\mathbb{R}/(2\pi\mathbb{Z})$, having absolutely convergent series of its Fourier

coefficients, equipping by the norm

$$\|f\|_{A(\mathbb{T})} := \sum_{k\in\mathbb{Z}} |\hat{f}(k)|$$

where

$$\hat{f}(k) = (2\pi)^{-1}\int_{\mathbb{T}} f(t)e^{-ikt}dt\,, \quad k \in \mathbb{Z}\,.$$

Let $\phi$ be a continuous map on $\mathbb{T}$ to itself, that is a continuous function $\phi : \mathbb{R} \to \mathbb{R}$ such that $\phi(t + 2\pi) \equiv \phi(t) \pmod{2\pi}$. We establish the following version of a well–known Beurling–Helson theorem, improving a recent result of V.V. Lebedev. **Theorem**: Let $\phi : \mathbb{T} \to \mathbb{T}$ be a continuous map. Suppose that

$$\|e^{in\phi}\|_{A(\mathbb{T})} = o\left(\frac{\log^{1/22}|n|}{(\log\log|n|)^{3/11}}\right)\,, \quad n \in \mathbb{Z}, \quad |n| \to \infty\,. \tag{1}$$

Then for some $\nu \in \mathbb{Z}$ the following holds $\phi(t) = \nu t + \phi(0)$. Let $p$ be a prime, $\mathbb{Z}_p = \mathbb{Z}/p\mathbb{Z}$, $A$ be a subset of $\mathbb{Z}_p$. Denote

$$\|\chi_A\|_{A(\mathbb{Z}_p)} = \frac{1}{p}\sum_{\xi\in\mathbb{Z}_p}\left|\sum_{a\in A}\exp(2\pi i a\xi/p)\right|.$$

To study $\|\chi_A\|_{A(\mathbb{Z}_p)}$, one can consider that $|A| \le p/2$. Trivially, $\|\chi_A\|_{A(\mathbb{Z}_p)} \ge 1$ for $|A| \ge 1$. For small $|A|$ we prove the best possible up to a constant lower estimate for $\|\chi_A\|_{A(\mathbb{Z}_p)}$ in terms of $|A|$. **Theorem** Let $p$ be a prime number, $A \subset \mathbb{Z}_p$, and

$$2 \le |A| \le \exp\left((\log p/\log\log p)^{1/3}\right).$$

Then $\|\chi_A\|_{A(\mathbb{Z}_p)} \gg \log|A|$. Moreover, we can estimate $\|\chi_A\|_{A(\mathbb{Z}_p)}$ from below nontrivially for all larger $A$, $|A| \le p/2$. Joint work with I.D. Shkredov.

**Van H. Vu** (Yale University). *Random walks and sumsets: Getting drunk is more dangerous than you may think.*
Abstract. Imagine a drunkard walking randomly on a grid. We know, from classical probability, that after n steps, his returning probability is of order $n^{-d/2}$, where $d$ is the dimension. We consider $d > 1$, omitting the boring case $d = 1$. This is monotone in $d$, and is "best" (in the drunkard's interest) if $d = 2$, as he has a good chance to return home. But what if the drunkard does not live in Manhattan? (While Manhattan has its share of drunkards, studies indicate that the distribution of drunkard is fairly uniform geographically.) It turns out, under a natural setting, that the two dimensional case is actually the absolute worst!! Our studies rely on very recent research on inverse theorems in probability, and it leads to new problems concerning geometrical incidences, which are of independent interest (being drunk or not).

**David J. Grynkiewicz** (University of Memphis). *The Index of Minimal Zero-Sum Sequences and Kummer Subspaces.*

Abstract. Let $n \geq 2$ be a fixed integer. Given an integer $x$, Let $(x)_n$ denote the least non-negative integer representative for $x$ modulo $n$. For an integer tuple $(x_1, x_2, x_3, x_4)$, let

$$||(x_1, x_2, x_3, x_4)||_n = \min\{(\alpha x_1)_n + (\alpha x_2)_n + (\alpha x_3)_n + (\alpha x_4)_n : \ gcd(\alpha, n) = 1\}.$$

We say that $(x_1, x_2, x_3, x_4)$ is a minimal zero-sum modulo $n$ if $x_1 + x_2 + x_3 + x_4 \equiv 0 \mod n$ and no proper sub-tuple has the sum of its coordinates congruent to 0 modulo $n$. It is an open conjecture that

$$||(x_1, x_2, x_3, x_4)||_n = n$$

for any minimal zero-sum tuple $(x_1, x_2, x_3, x_4)$. This has been verified, among other cases, when $n$ has at most two distinct prime divisors. In this talk, we present a new perspective for the above conjecture, explain how this leads to simplified proofs for known cases in the conjecture, and then briefly go over how such results can be applied to classify monomial Kummer subspaces in central simple algebras of degree $p$. Joint work with A. Chapman, E. Matzri, L. H. Rowen and U. Vishne.

**Yonutz V. Stanchescu** (Afeka Academic College and Open University). *Discrete and continuous Bonnesen-Minkowski type inequalities.*

Abstract. The Brunn-Minkowski Theorem asserts that $\mu_d(A + B)^{1/d} \geq \mu_d(A)^{1/d} + \mu_d(B)^{1/d}$ for convex bodies $A$, $B \subseteq \mathbb{R}^d$, where $\mu_d$ denotes the $d$-dimensional Lebesgue measure. Let $H$ be a hyperplane. Bonnesen strengthened this bound by showing

$$\mu_d(A + B) \geq (M^{1/(d-1)} + N^{1/(d-1)})^{d-1}(\frac{\mu_d(A)}{M} + \frac{\mu_d(B)}{N}),$$

where $M = \mu_{d-1}(\pi(A))$ , $N = \mu_{d-1}(\pi(B))$ and $\pi$ denotes a projection of $\mathbb{R}^d$ onto $H$. We characterize the cases of equality in this later bound, showing that equality holds if and only if $A$ and $B$ are obtained from a pair of homothetic convex bodies by 'stretching' along the direction of the projection. We obtain a similar result for two dimensional *finite* sets $A$, $B \subseteq \mathbb{R}^2$, which are extremal for the discrete version of Bonnesen's inequality: $|A + B| \geq (\frac{|A|}{m} + \frac{|B|}{n} - 1)(m + n - 1)$, where $m$ and $n$ are the minimum number of parallel lines covering $A$ and $B$ respectively. Joint work with G. A. Freiman, D. J. Grynkiewicz and O. Serra.

**Oriol Serra** (Universitat Politecnica de Catalunya, Barcelona, Spain). *On Sumsets and Convex Hull.*

Abstract. One classical result of Freimann gives the optimal lower bound for the cardinality of $A + A$ if $A$ is a $d$-dimensional finite set in Euclidian space. Matolcsi and Ruzsa have recently generalized this lower bound to $|A + kB|$ if $B$ is $d$-dimensional and $A$ is contained in the convex hull of $B$.

We characterize the equality case of the Matolcsi-Ruzsa bound. The characterization combines geometric and arithmetic structure. Joint work with K. Boroczky and P. Santos.

**Vsevolod F. Lev** (Haifa University). *Approximate Convexity and an Edge-Isoperimetric Estimate.*
Abstract. Let $\mathcal{F}_0$ denote the class of all real-valued functions defined on the interval $[0,1]$, and satisfying the weak convexity condition

$$f(tx+(1-t)y) \le tf(x)+(1-t)f(y)+|y-x|,\ t,x,y\in[0,1]$$

and the boundary condition

$$\max\{f(0),f(1)\}\le 0.$$

It is not difficult to see that the pointwise supremum $F := \sup\{f\colon f\in\mathcal{F}_0\}$ is itself a function from $\mathcal{F}_0$. Can this function be found explicitly? We answer this question showing that

$$F(x)=\min\{k\|x\|^{1-1/k}\colon k\ge 1\},\ x\in[0,1],$$

where $\|x\| = \min\{x, 1-x\}$. As a (somewhat unexpected) application, we prove that if $A$ and $S$ are subsets of a finite abelian group $G$, such that $S$ is generating and all of its elements have order at most $m$, then the number of edges from $A$ to its complement $G\setminus A$ in the directed Cayley graph induced by $S$ on $G$ is at least $m^{-1}F(\alpha)|G|$, where $\alpha := |A|/|G|$ is the density of $A$ in $G$.

**Melvyn B. Nathanson** (City University of New York). *A problem of Rankin on sets without geometric progressions.*
Abstract. A *geometric progression of length k and integer ratio* is a set of numbers of the form $\{a, ar, \ldots, ar^{k-1}\}$ for some positive real number $a$ and integer $r\ge 2$. For each integer $k\ge 3$, a greedy algorithm is used to construct a strictly increasing sequence $(A_i)_{i=1}^{\infty}$ of positive integers with $A_1 = 1$ such that the set $G^{(k)} = \bigcup_{i=1}^{\infty}\left(\frac{1}{A_{2i}}, \frac{1}{A_{2i-1}}\right]$ contains no geometric progression of length $k$ and integer ratio. Moreover, $G^{(k)}$ is a maximal subset of $(0,1]$ that contains no geometric progression of length $k$ and integer ratio. The set $G^{(k)}$ gives a new lower bound for the maximum cardinality of a subset of the set of integers $\{1,2,\ldots,n\}$ that contains no geometric progression of length $k$ and integer ratio. Joint work with Kevin O'Bryant.

**Gregory A. Freiman** (Tel-Aviv University). *An exact volume estimate for sets of integers with a given doubling constant.*
Abstract. It is known that any set of integers A for which —A+A— is at most c—A— lies in a low dimensional generalized arithmetic progression of small volume. In a series of papers by Bilu, Ruzsa, Mai-Chu Chang, Konyagin, Sanders and others estimates of this volume have been obtained and improved. In this talk a new way to get exact estimates will be discussed,

accompanied by some examples.

## 16. Teaching with Mathematical Habits in Mind

**Davida Fischman** (California State University, San Bernardino). *An overview and synthesis of the Habits of Mind literature for mathematics education.*
Abstract. In this talk we will present an overview of the different conceptualizations of habits of mind relevant to mathematics teaching and learning, and provide a framework that synthesizes some of these various conceptualizations. This talk is meant to frame the rest of the presentations in this special session. Joint work with Jennifer Lewis.

**Paul Goldenberg** (Education Development Center). *Building the disposition to seek mathematical structure and to puzzle through problems.*
Abstract. People treat puzzles and problems quite differently. Problems are often perceived as mere exercises, a consequence of schooling in which the exercises match the content and techniques of the chapter and very few (or none) require the student to figure out how to start. As a result, when students encounter a problem that they don't very quickly see how to start and what to do, either they feel bad at math or they complain that the teacher has not taught them how or feel that the problem is a trick. Puzzles are different. Not knowing where to start is part of what makes it a puzzle, and so we give ourselves a bit more breathing room to search the structure and clues, and don't so quickly conclude we don't know enough. Even in a crossword puzzle, we know that we may need to search many dozens of clues before we find anything useful, and may search the structure of the puzzle to help us look for likely clues. We also use the interaction of the clues. That's exactly the disposition we'd like students to bring to their mathematical problems. And that's also the nature of real life: real problems in real life just occur, without asking what chapter we've just studied. Suitably chosen classroom problems and puzzles can, unlike a crossword puzzle, be vehicles for the relevant mathematical content and can, unlike mere exercises, build some of the mathematical habits of mind that problem-solvers and system builders require. This session will focus on both traditional problems and puzzle-types chosen with this goal.

**Nitsa Movshovitz-Hadar** (Technion). *Is an expository lecture bound to jeopardize the development of habits of mind?.*
Abstract. In the past two decades, developing mathematical habits of mind has become an almost synonym to the ultimate goal of good mathematics education. Learning mathematics by doing mathematics has become the ultimate response to the question – how to do it, which occupies mathematics educators all through elementary, secondary and tertiary education,

whereas giving an expository lecture has become an almost "dirty word". In my talk I would like to tackle a provocative question: An expository presentation of mathematical ideas, does it jeopardize the development of habits of mind? or can it possibly support it? – Findings from two independent analyses will be presented:
(i) An analysis of a series of videotaped lectures in Algebra given by a permanent best lecturer awardee at Technion – David Tsilag, who passed away last year, but his lectures keep attracting a huge number of visitors on YouTube (e.g. `youtube.com/watch?v=kTSZuTX0zng`);
(ii) An analysis of the "big ideas" threaded across a new initiative to expose high school students to Mathematics News Snapshots, despite their lack of sufficient preparation to delve into the depth of contemporary mathematics. (See a sample snapshot `edu.technion.ac.il/docs/sudoku.ppsx`).
My claim is that expository presentations of these sorts support the development of students' mathematical habits of mind rather than jeopardize it. What is it that makes a mathematical exposition support such a development, will be discussed.

**Avraham Arcavi** (Weizmann Institute). *Teaching mathematics having in mind habits of mind.*
Abstract. I will describe main characteristics of elementary school mathematics lessons as I experienced them first hand during my stay in Japan. I will provide examples of mathematical tasks, student-teacher interactions, and teacher decision making, stressing the mathematical and pedagogical considerations at stake.

**Ruhama Even** (Weizmann Institute). *Factors involved in shaping students' opportunities to develop habits of mind in the context of school mathematics.*
Abstract. In this talk, I will examine the ways by which key factors in school mathematics – the mathematics content, the textbook, the teacher, and the class – shape students' opportunities to develop habits of mind. Drawing on several studies conducted by my research group I will demonstrate the critical role of each factor as well as the importance of attending to their interplay.

**Gerald A. Goldin** (Rutgers University). *Affective structures, engagement, and mathematical habits of mind.*
Abstract. To understand in-the-moment mathematical problem solving activity, the description of cognitive structures and how they develop has long been seen as a valuable approach. Analogously, to understand the nature of mathematical engagement and the development of fruitful habits of mind, it is worthwhile to consider affective structures involving emotions, attitudes, beliefs, and values, that enable ways of responding in particular contexts. More specifically, what we have termed engagement structures are idealized

patterns of thought, emotion and behavior, including social interactions, evoked by particular motivating desires in mathematical contexts. This talk describes some of the underlying ideas, focusing on aspects that are to some degree domain-specific to mathematics.

**Boris Koichu** (Technion). *On developing habits of mind associated with mathematical problem solving: Lessons from a series of (relatively) long-term intervention studies with middle and high school students.*
Abstract. Can specially designed classroom and out-of-classroom problem-solving activities lead to the development of useful habits of mind in middle-school students? I argue that in some cases, for some students and in relation to particular habits it can happen as a result of long-term effort. The argument is based on examples from four intervention studies with middle and high school students. In the first study, the practice of using heuristic vocabulary in problem-solving discourse was facilitated in two 8th grade classes during a five-month period. This practice turned to be a useful habit of mind mostly for those participants who were characterized as weak problem solvers at the beginning of the study. In the second study, the practice of systematic search for deep-level connections between solved and to-be-solved problems was in focus in 10th and 11th grade classes. The usefulness of this practice became apparent to the students only towards the end of a year-long intervention. Some participants began using it voluntarily, which provided us with the evidence that search for deep-level connections among problems became a habit of mind for them. In the third study, about 40 small teams of 9th grade students were engaged in doing mathematical research for about four months. In several teams, the students developed powerful strategies of inventing, pattern-sniffing and generalizing and began systematically use these strategies in different problem-solving situations. In the fourth (ongoing) study, we experiment, in a 9th grade class, with open-ended problem solving in a learning environment combining a classroom collaborative setting, individual work and on-line asynchronous forums. Preliminary data analysis suggests that some of the students developed an effective habit of successful overcoming anxiety by smartly combining individual and collaborative effort. The talk is concluded by consideration of the lessons learned through the lenses provided by an emerging confluence model of learning through mathematical problem solving.

**Jennifer Lewis** (Wayne State University). *Defining and measuring Proclivities for Teaching Mathematics.*
Abstract. This paper presents a conceptual framework that we call proclivities for teaching mathematics. We define mathematical proclivities as tendencies to action in teaching mathematics, guided by habits of mind. This framework grows from the literatures on habits of mind and critical thinking and is informed by current policy directions in mathematics education. We argue that the notion of mathematical habits of mind has been

both overused and underspecified in mathematics education, and we show how what we call mathematical proclivities operate at three distinct levels in the work of teaching. The paper presents data from a set of piloted measures that are designed to give the outlines of this conceptualization of teacher thinking, and we show how these proclivities are associated with other measures of teacher performance. Joint work with Davida Fischman.

**Theodore Eisenberg** (Ben-Gurion University). *Concluding remarks about the Special Session on Habits of Mind.*
Abstract. In a participatory format, the chairs of this Special Session will engage all participants in conversation about the themes that cut across the papers presented during the conference. Joint work with Davida Fischman and Jennifer Lewis.

## 17. Combinatorial Games

**Thane Plambeck** (Counterwave, Inc., Palo Alto, California). *Misere Quotients for Beginners.*
Abstract. We'll summarize some particularly simple techniques for the analysis of misere-play impartial combinatorial games that don't involve a full-blown deployment of the misere quotient theory, in the hopes that more "normal-play only" analysts might be tempted to apply them to the (many) misere versions of impartial games that no one has looked at so far.

**Lowell Abrams** (George Washington University). *Almost-symmetry for a family of Nim-like arrays.*
Abstract. For multiple-pile blocking Nim, the mex rule applies symmetrically, and if positions are blocked symmetrically as well, then the symmetry of P-positions is ensured. What happens, though, if the blocking is not done symmetrically? We will present some results exhibiting some "almost symmetric" games, and discuss connections with scaling-by-2 symmetry that seems to be independent of the symmetry of P-positions.

**Aviezri S. Fraenkel** (Weizmann Institute). *Games, Numeration Systems and Data Structures.*
Abstract. A primary aim of combinatorial game theory is to formulate tractable winning strategies for games. I wish to sell you the idea that sometimes the only known method to do so is via a judiciously chosen numeration system, analogously to the choice of an appropriate data structure for optimization problems. Time permitting, we will also see that numeration systems may be conducive to solve elegantly other problems in math.

**Alex Fink** (Queen Mary University of London, United Kingdom). *The generating function method for impartial games.*
Abstract. It is possible to solve some impartial games purely algebraically, by manipulating the generating function for the set of P positions. I demonstrate this for Nim, Lim, Welter's game, and subtraction games whose subtraction set is an arithmetic progression. Though each case requires a different variation on the method, the key behaviour the games share allowing it is that they have, or can be transformed to have, an odd number of P options from each N position. Are there any yet-unsolved games out there that yield to this method?

**Urban Larsson** (Chalmers University of Technology, Sweden). *Scoring Combinatorial Game Theory, A Useful Universe: Part I.*
Abstract. Finding general results for scoring games has proven difficult. Ettinger, Johnson, Milnor, and Stewart considered sub-classes with varying degrees of success. We introduce a general class of scoring games that is a subclass of those considered by Stewart, and that naturally extends Conway's Normal-play games. We present a situation, an obstacle, that if allowed in the class prevents there being an order-preserving map of Normal-play games into the class. Our game universe is closed under disjunctive sum, has unique canonical forms and several other desirable properties, at least from a game-players point of view. (Part II will be given by Carlos P. Santos) Joint work with Carlos P. Santos, University of Lisbon; Richard J. Nowakowski, Dalhousie University, Nova Scotia.

**Carlos Santos** (Center for Linear Structures and Combinatorics, University of Lisbon, Portugal). *Advances on Scoring Combinatorial Game Theory.*
Abstract. Finding general results for scoring games has proven difficult. Ettinger, Johnson, Milnor, and Stewart considered sub-classes with varying degrees of success. We introduce a general class of scoring games: that is a subclass of those considered by Stewart; that naturally extends Conway's Normal-play games; and that also includes the classes considered by Ettinger and Milnor. We show that this class has the desirable properties We present a situation, an obstacle, that if allowed in the class prevents there being an order-preserving map of Normal-play games into the class. We present a framework that naturally extends both Conway's normal play games and Ettinger's dicot games, that is closed under disjunctive sum, has unique canonical forms and has several other desirable properties, at least from a game-players point of view. Joint work with Richard Nowakowski, Dalhousie University, Nova Scotia, Canada; Urban Larsson, Chalmers and University of Gothenburg.

**Solomon Golomb** (University of Southern California). *Tic-Tac-Toe in N Dimensions.*
Abstract. We study the analogue of tic-tac-toe played on a $k$-dimensional

hypercube of side $n$. The game is either a first-player win or a draw. We are primarily concerned with the relationships between $n$ and $k$ (regions in $n-k$ space) that correspond to wins or draws of certain types. For example, for each given value of $k$, we believe there is a critical value $n_{dd}$ of $n$ below which the first player can force a win, while at or above this critical value, the second player can obtain a draw. The larger the value of n for a given $k$, the easier it becomes for the second player to draw. We also consider other "critical values" of $n$ for each given $k$ separating distinct behaviors. Finally, we discuss and prove results about the misere form of the game. Joint work with Alfred W. Hales, Center for Communication Research, San Diego.

**Matthieu Dufour** (University of Quebec at Montreal, Canada). *Circular Nim games.*
Abstract. Circular Nim, a variation on the well-known game of Nim, is a two-player impartial combinatorial game consisting of $n$ stacks of tokens placed in a circle. A move consists of choosing $k$ consecutive stacks, and then taking at least one token from one or more of the $k$ stacks. Typical questions are whether there is a strategy for one of the two players that allows this player to win no matter how the other player plays. This question is answered by determining the set of losing positions.
I will give an overview on how to find/guess the set of losing positions and present results we have obtained on the structure of the losing positions for circular Nim. For $n \leq 6$, all but the game for $n = 6$ and $k = 2$ are solved, and we have also solved the game for the case $n = 8$ and $k = 6$. We will discuss the difficulties that arise with the still open question when $n = 6$ and $k = 2$. Joint work with Silvia Heubach, California State U.

**Eric Duchene** (Lyon 1 University, France). *Invariant games.*
Abstract. Invariant games are games for which the set of available moves does not depend on the current position. They were introduced in 2010 and until now, are studied in the case of impartial removal games on heaps. For example, Nim and (generalized) Wythoff are invariant. The Tribonacci game, the Flora game, or the Raleigh game are not invariant. An interesting question recently posed is the following: given a sequence $S$ of P-positions, does there exist a game whose set of P-positions exactly corresponds to $S$? The answer is yes if we do not constrain this game to be invariant. If we search for an invariant game, then the answer is not obvious. Fraenkel et al. proved that an invariant game exists for any Beatty sequence coding the P-positions. We extended it to non-homogeneous Beatty sequences. In this talk we will present the main results about invariant games. In addition, we will code the sequence of P-positions by words (which generalizes algebraic formulas), and see that for a large set of words, one can decide the existence of an invariant game in polynomial time.

**Urban Larsson** (Chalmers University of Technology, Sweden). *Combinatorial Number Theory and Wythoff Nim Extensions.*
Abstract. What is the minimal lower density of a subset of the natural numbers, given a certain 'distance' to its complement set? We study number theoretical aspects for extensions of the classical impartial combinatorial game of Wythoff Nim. The games are played on two heaps of tokens, and have 'symmetric' move options. We show that the inverse of the Golden ratio is the minimal lower asymptotic density for the x-coordinates of a given game's 'upper' P-positions. As an example of the powerfulness of this result, we study a game in the family a Generalized Diagonal Wythoff Nim, recently introduced by Larsson. A certain split of P-positions, distributed in a number of so-called P-beams, was conjectured for many such games. The term split here means that an infinite sector of upper positions is void of P-positions, but with infinitely many upper P-positions above and below it. By using the first result, we prove this conjecture for one of these games, called $(1, 2)$-GDWN, where a player moves as in Wythoff Nim, or instead chooses to remove a positive number of tokens from one heap and twice that number from the other.

**Silvia Heubach** (California State University Los Angeles). *Building Nim.*
Abstract. The game of Nim, with its simple rules, its elegant solution and its historical importance is the quintessence of a combinatorial game, which is why it led to so many generalizations and modifications. We present a modification with a new spin: "Building Nim". With n tokens and s stacks, this two-player game is played in two phases: first, the building phase, where players alternate to put one token on one of the s (initially empty) stacks until all n tokens have been used. Then, the players (beginning with the player who started the building phase) simply play Nim. Of course, because the solution for the game of Nim is known, the goal of the first player is a placement of the tokens so that the digital sum of the s stack heights at the end of the first phase is different from zero, while the goal of the second player is to make this sum equal to zero.
This game is a trivial win for the first player if n is odd as the digital sum could never be zero. It is also a win for the second player if both n and s are even as the second player simply copies the first players move on another stack, resulting in a digital sum of zero after each of the second player's moves, and therefore, at the end of the first phase. We present a partial solution for this game for the non-trivial cases, namely when n is even and s is odd. Specifically, we give results for $s = 3$, $s = 5$, and for $s > 5$, $n \leq s + 3$. Joint work with Eric Duchene, Claude Bernard University of Lyon 1; Matthieu Dufour, University of Quebec at Montreal; Urban Larsson, Chalmers and University of Gothenburg.

## 19. Applications of Algebra to Cryptography

**Tali Kaufman** (Bar-Ilan University). *High Dimensional Expanders and Property Testing.*
Abstract. We show that the high dimensional expansion property as defined by Gromov, Linial and Meshulam, for simplicial complexes is a form of local testability, a well studied concept in the theory of computing. Namely, a simplicial complex is a high dimensional expander iff a suitable property is locally testable. Using this connection, we derive several testability results. Joint work with Alex Lubotzky (HUJI).

**Enric Ventura** (Universitat Politècnica de Catalunya, Spain). *Commuting degree of infinite groups.*
Abstract. There is a classical result saying that, in a finite group, the probability that two elements commute is never between 5/8 and 1 (i.e., if it is bigger than 5/8 then the group is abelian). It seems clear that this fact cannot be translated/adapted to infinite groups, but it is possible to give a notion of commuting degree for finitely generated groups (w.r.t. a fixed finite set of generators) as the limit of such probabilities, when counted over successively growing balls in the group. This asymptotic notion is a lot more vague than in the finite setting, but we are still able to prove some results concerning this new concept.

**Arkadius Kalka** (Bar-Ilan University). *Simultaneous conjugacy and double coset problem in braid and Garside groups.*
Abstract. We solve the simultaneous conjugacy problem (SCP) in Garside groups, by means of an effectively computable invariant. This invariant generalizes the notion of super summit set of a conjugacy class. One motivation for studying the SCP is that the security of a number of proposals of cryptographic key exchange protocols in non-commutative cryptography can be reduced to the Search SCP in groups. Another motivation for the SCP is that a number of computational problems in braid groups reduce to it. In particular, we show that the Double Coset Problem for parabolic subgroups of braid groups (with connected associated Coxeter graph) reduces to the SCP.

**Boaz Tsaban** (Bar-Ilan U and Weizmann Inst). *Polynomial time solutions of the main problems of noncommutative algebraic cryptography.*
Abstract. Around the year 2000, original proposals for cryptographic key exchange protocols based on Artin's braid groups led to a lot of excitement within the mathematical community. Heuristic cryptanalyses of these protocols suggested that using specially crafted distributions would render them secure.
We have recently found provable, polynomial time algorithms for solving

the underlying computational problems. This provides an unexpected conclusion of the above-mentioned program.
We will describe the protocols, the underlying computational problems, and their polynomial time solutions. The talk will be mostly based on out paper `dx.doi.org/10.1007/s00145-013-9170-9`. We may also describe a recent improvement, arrived at jointly with Avraham (Rami) Aizenbud of WIS.

**Delaram Kahrobaei** (City University of New-York). *A CCA secure cryptosystem using matrices over group rings.*
Abstract. We propose a cryptosystem based on matrices over group rings and claim that it is secure against adaptive chosen ciphertext attack. Joint work with C. Koupparis and V. Shpilrain.

**David Garber** (Holon Institute of Technology). *Analyzing the length-based attack on polycyclic groups.*
Abstract. After the Anshel-Anshel-Goldfeld (AAG) key-exchange protocol came out in 1999, it was studied with braid groups and then with Thompson's group as the underlying platform. The length-based attack, first originated by Hughes and Tannenbaum, has been used to extensively study AAG with the braid group platform. Meanwhile, a new platform, using polycyclic groups, was proposed by Eick and Kahrobaei. In this talk, we present this platform, and we claim that with a high enough Hirsch length, the polycyclic group as a platform for AAG is resistant to the known various versions of the length-based attack. Joint work with Delaram Kahrobaei (CUNY) and Ha T. Lam (CUNY).

**Andrzej Zuk** (Paris 7, France). *Random walks on symmetric groups.*
Abstract. We present combinatorial and algorithmic aspects of random walks on symmetric groups. Joint work with H. Helfgott (ENS Paris).

**Inna Bumagin** (Carleton University, Canada). *Conjugacy problem in relatively hyperbolic groups.*
Abstract. I will discuss a polynomial time algorithm to solve the conjugacy problem and the multiple conjugacy problem for hyperbolic elements in relatively hyperbolic groups.

## 20. Topological Graph Theory and Map Symmetry

**Martin Škoviera** (Comenius University, Bratislava). *Hamilton cycles in truncated triangulations of closed surfaces.*
Abstract. A truncated triangulation is a 3-valent map on a closed surface $S$ arising from a triangulation of $S$ by truncating each vertex, that is, by expanding it into a contractible cycle. Several years ago, Glover and Marušič

implicitly employed truncation of highly symmetrical triangulations of orientable surfaces as a tool for proving the existence of Hamilton cycles in large classes of cubic Cayley graphs. We generalise their method by replacing high symmetry with a somewhat surprising weaker condition – upper embeddability of the dual cubic graph. Our result enables us to construct Hamilton cycles in much wider classes of cubic graphs. For example, we show that the truncation of a triangulation of any closed surface with no separating 3-cycle has a Hamilton path and has a Hamilton cycle whenever the number of faces is $2\,(mod\,4)$ Jones. The case $0\,(mod\,4)$ will be discussed in the subsequent talk of Roman Nedela.Joint work with Michal Kotrbčík and Roman Nedela.

**Matej Roman Nedela** (Bel Univ., Banská Bystrica). *Cycles in truncated triangulations missing at most two vertices.*
Abstract. A truncated triangulation is a 3-valent map on a closed surface $S$ arising from a triangulation $T$ of $S$ by truncating each vertex, that is, by expanding it into a contractible cycle. In the previous talk, presented by Martin Škoviera, we have discussed the existence of Hamilton cycles in the truncation of $T$ and described a rather surprising relationship to upper-embeddability of the underlying graph of the dual cubic map $T^*$. The method provides a Hamilton path in general and a Hamilton cycle whenever the number of triangles of $T$ is $2\,(mod\,4)$; otherwise no long cycle in the truncation is guaranteed. To force a long cycle in the truncation when the number of triangles of $T$ is $0\,(mod\,4)$ we need a stronger version of upper-embeddability – ample upper-embeddability – of a cubic graph. This stronger concept is very natural and guarantees the existence of a cycle missing only two adjacent vertices in the truncation of $T$. Understanding which cubic graphs are amply upper-embeddable is an interesting problem on its own. There exist infinite families 2-connected upper-embeddable cubic graphs that are not amply upper-embeddable. On the other hand, we show that all cyclically 4-connected cubic graphs are amply upper-embeddable, strengthening a classical result of Payan and Xuong (1979). Joint work with Michal Kotrbčík and Martin Škoviera.

**Jozef Širáň** (Open U and Slovak U of Tech). *Chiral regular maps of a given type.*
Abstract. With the help of representation theory and holomorphic differentials on Riemann surfaces, G. A. Jones recently proved the existence of a chiral orientably regular hypermap of any given hyperbolic type. We present an alternative proof of this result for maps, using Jones' more general version of Jordan's lemma on primitive permutation groups containing a cycle and lifts of planar tree-like maps. Joint work with Marston D.E. Conder, Veronika Hucíková and Roman Nedela.

**Gareth A. Jones** (University of Southampton, UK). *Chirality of hypermaps and dessins.*
Abstract. Hypermaps, which can be regarded as bipartite graphs embedded in surfaces, are combinatorial representations of Grothendieck's dessins d'enfants. By Belyĭ's Theorem they link the Galois theory of algebraic number fields and the Teichmüller theory of Riemann surfaces. Grothendieck [1] noted that they give a faithful representation of the absolute Galois group $\Gamma = \mathrm{Gal}\overline{\mathbb{Q}}/\mathbb{Q}$, a profinite group which played a major role in the proof by Wiles and Taylor of Fermat's Last Theorem. A recent result of González-Diez and Jaikin-Zapirain shows that this action of $\Gamma$ remains faithful when restricted to regular dessins. These are the most symmetric dessins, corresponding to the orientably regular hypermaps, a natural generalisation of the Platonic solids regarded as maps on the sphere.
It is unknown whether regular dessins are typically symmetric (isomorphic to their mirror images) or chiral: computational searches and theoretical results give insufficient evidence. In 1992 Singerman [3] conjectured that whenever $\frac{1}{m} + \frac{1}{n} \leq \frac{1}{2}$ there exists an orientably regular chiral map of type of valency $n$. Hucíková, Nedela and Širáň [4] have recently announced a proof of this conjecture. The aim of this talk is to present a similar but more general result [2] for hypermaps (and hence for dessins):
Theorem: *There exist infinitely many orientably regular chiral hypermaps of each non-spherical type.*

The construction applies representation theory to the actions of automorphism groups of surfaces on spaces of differentials and on homology groups.
References:
[1] A. Grothendieck, Esquisse d'un Programme, in *Geometric Galois Actions I, Around Grothendieck's Esquisse d'un Programme* (ed. P. Lochak and L. Schneps) London Math. Soc. Lecture Note Ser. 242 (Cambridge University Press, Cambridge, 1997), pp. 5–48.
[2] G. A. Jones, Chiral covers of hypermaps, arXiv.math:1311.4355, 2013.
[3] D. Singerman, Reflexibility and symmetry, preprint, University of Southampton, 1992.
[4] J. Širáň, How symmetric can maps on surfaces be?, in *Surveys in combinatorics 2013*, London Math. Soc. Lecture Note Ser. 409 (Cambridge University Press, Cambridge, 2013), pp. 161–238.

**Gadi Moran** (Haifa University). *The conjugacy semigroups of the symmetric groups and maps on orientable surfaces.*
Abstract. A brief review of the topic will be offered. *The Conjugacy Semigroup over a group G* is the family of its subsets which are unions of conjugacy classes, with product-of-subsets as the semigroup operation *Bicolored Maps on Orientable Surfaces* arise naturally in the study of the product of two conjugacy classes of Symmetric groups. A notion of *parity for some conjugacy classes whose members move infinitely many elements* arises also

in this study.

References (consult citation therein for more):
[1] Conjugacy Classes whose square is an Infinite Symmetric Group, Trans. A.M.S 316 (2) (1989) 493–522.
[2] The Products of Conjugacy Classes in some Infinite Simple Groups, Israel J. Math 50 (1985).
[3] Parity Features for Classes of the Infinite Symmetric Group, J. Combin. Theory A 33 (1982) 82–98.
[4] Products of involution classes in Infinite Symmetric Groups, Trans. A.M.S 307 (1988) 745–762.

**Mark E. Watkins** (Syracuse University). *Graphical Frobenius representations of abstract groups.*
Abstract. A group of permutations acting transitively on an $n$-set is regular if every nonidentity element of the group has no fixed point; it is a Frobenius group if it is transitive and only the identity fixes more than one point. A classic theorem of Frobenius states that the $n-1$ fixed-point-free elements of a Frobenius group $G$ together with the identity form a normal, regular subgroup of $G$, called the Frobenius kernel. A problem solved over the period between 1958 and 1982 was to characterize the finitely generated abstract groups $G$ admitting a graphical regular representation (GRR), namely a graph $\Gamma$ whose automorphism group is isomorphic to $G$ and acts as a regular permutation group on the vertex set of $\Gamma$, in which case $\Gamma$ is necessarily a Cayley graph for $G$. Here we seek those abstract groups $G$ admitting a graphical Frobenius representation (GFR), namely a graph $\Gamma$ whose automorphism group is isomorphic to $G$ but whose action is that of a Frobenius group. In this case, $\Gamma$ must be a Cayley graph for the Frobenius kernel. We give some necessary and some sufficient conditions for a group to admit a GFR. Various families of normal products of an odd Abelian group by a subgroup of its automorphism group admit a GFR. A sample of groups that do not includes those with nontrivial centers, all nontrivial direct products, and all groups of odd cube-free order. All groups of order $\leq 300$ that admit a GFR are determined. Joint work with John Kevin Doyle and Thomas W. Tucker.

**Jonathan L. Gross** (Columbia University). *Interpolation for partial genus distributions.*
Abstract. The *genus distribution* of a graph $G$ is the sequence $\langle g_i(G) \mid i \geq 0 \rangle$, where $g_i(G)$ denotes the number of cellular imbeddings of a graph $G$ into the oriented surface $S_i$. This investigation is part of an extensive research program directed toward the conjecture that the genus distribution of every graph is log-concave. One of the most productive ways to calculate a genus distribution has been to partition it into *partial genus distributions* according to the incidences of face-boundary walks on root-edges or root-vertices.

This is the first investigation specifically directed at the nature of *partial genus distributions*. Since the largest subscript $i$ for which $g_i(G)$ is non-zero has an upper bound of $\lfloor \frac{\beta(G)}{2} \rfloor$, it follows that there are only finitely many non-zero values for $g_i(G)$ or for any partial genus distribution. A sequence is said to have the *interpolation property* or to be *interpolating* if it has no internal zeros. It has been known for half a century, by Duke's interpolation theorem, that the sequence $\langle g_i(G) \mid i \geq 0 \rangle$ has the interpolation property. We prove here that various partial genus distributions have the interpolation property. Toward this objective, we introduce two new methods, one called *iterated $r$-move surgery*, and the other called *double rotational surgery*. Joint work with Toufik Mansour and Thomas W. Tucker.

**Toufik Mansour** (Haifa University). *Log-concavity of the genus polynomials for a sequence of cubic Halin graphs.*
Abstract. A *Halin graph* is a graph obtained from a plane tree by running a cycle through its leaf vertices in the order they are encountered along a counterclockwise pre-order traversal. Using a *vectorized production matrix*, we give a matrix formula for the *partitioned genus polynomial* of any cubic Halin graph and for the *genus polynomial* as well. We prove log-concavity of the genus polynomial and of the partitioned genus polynomials for several sequences of cubic Halin graphs, which serves as further support of the conjecture that the genus polynomial of every graph is log-concave. Joint work with Jonathan L. Gross and Thomas W. Tucker.

**Thomas W. Tucker** (Colgate University). *The imbedding-type module for the genus distribution of a graph.*
Abstract. The *genus distribution* of a graph $G$ counts, for each $i$, the number of imbeddings of $G$ in the orientable surface of genus $i$. Since the total number of imbeddings of a graph is so large (e.g., $(9!)^{10}$ for $K_{10}$)), most of the graphs for which the genus distribution have been calculated are what Stahl called "linear" families, which have a natural recursive structure wherein a member is obtained from the previous one by attaching a copy of the same base graph along edges joining specified root vertices. In this talk, developing a suggestion of Mohar, we present a general algebraic structure, the *imbedding type module $M$*, where adding an edge between root vertices gives a linear transformation of $M$. The module has a basis of *imbedding-types*. Each type is a set of cyclic lists of root vertices corresponding to the faces of an imbedding, and the coefficients are the partial genus distribution (pgd) polynomials that serve as generating functions counting the number of imbeddings of a given type, according to genus. The imbedding-types have a natural ordering, analogous to Gröbner bases, that allows machine computation of *transfer matrices* for the recursion underlying a linear family. Until now, such matrices have been computed laboriously by hand, for only a few simple linear families. In addition to generating data to analyze for possible

properties of genus distributions (such as long-concavity), the imbedding-type module also provides a setting for studying the general structure of all genus polynomials. Joint work with Jonathan L. Gross and Toufik Mansor.

**David G.L. Wang** (Haifa University). *The root geometry of polynomials from recurrences of order two.*
Abstract. This talk reports recent progress on the root locations of polynomials coming from topological graph theory. We focus on polynomials defined by a recurrence of order two with unit-variable functional coefficients. We give sufficient conditions for real-rootedness, and determine the precise root intervals. I will also show some applications of the root distribution results for examples from combinatorics. Joint work with Jonathan L. Gross, Toufik Mansour and Thomas W. Tucker.

**Chen Yichao** (Hunan University). *On the mode of the genus distribution and the average genus for some types of graphs.*
Abstract. A real polynomial $f(x) = \sum_{i=0}^{n} a_i x^i$ is *unimodal* if there exists an index $j$, with $0 \leq j \leq n$, such that $a_0 \leq a_1 \leq \ldots \leq a_j \geq a_{j+1} \geq \ldots \geq a_n$, in which case $j$ is called a *mode* of the sequence. The *genus polynomial* of a graph $G$ is the polynomial $g_G(x) = \sum_{i=0}^{\infty} g_i x^i$, where $g_i$ is the number of embeddings, for $i = 0, 1, \ldots$, into the orientable surface $S_i$. The sequence $\{g_i(G)|i \geq 0\}$ is called the *genus distribution* of the graph $G$. The *average genus* $\gamma_{avg}(G)$ of a graph $G$ is the expected value of the genus random variable, over all labeled 2-cell orientable embeddings of $G$, using the uniform distribution. In this talk, we will discuss the mode of genus distribution and the average genus for some classes of graphs. Our calculation shows that the mode of the genus distribution of any of these types of graphs equals the upper-rounding or lower-rounding of its average genus.

**Jin Ho Kwak** (BJTU-China and POSTECH-Korea). *Genera of Cayley maps.*
Abstract. The genus distribution of a graph $G$ is defined to be the sequence $\{g_m\}$, where $g_m$ is the number of different embeddings of $G$ in the closed orientable surface of genus $m$. In this paper, we examine the genus distribution of Cayley maps for several Cayley graphs. Joint work with Jian-Bing Liu.

**Eunice Mphako-Banda** (University of the Witwatersrand). *The link component number of vertex-joins of certain graphs.*
Abstract. Links are fascinating geometrical objects. At the same time they are very simple to visualise and yet remarkably hard to analyse. In mathematics, we look at knotted and unknotted loops of string and analyse the forms and relationships between them. Links can be analysed using different tools in topology, group theory, algebra and graph theory. One problem in knot theory which is still of interest is experimenting with transferring

techniques in graph theory into knot theory. It has been shown in the literature that each signed planar graph has a corresponding link diagram and vice-versa. Thus each planar graph which is not signed has a corresponding link universe. This provides a bridge between knot theory and graph theory. There are many notions of graphs which have been transferred successfully into knots (links), one example is graph polynomials. In this talk we look at some outer planar graphs whose number of components of the corresponding links are known, then we form vertex joins of these graphs and count the number of components of the resulting corresponding links.

**Eli Berger** (Haifa University). *The algebraic and topological properties of claw-free graphs.*

Abstract. We study two parameters that can be associated with a given graph. The first parameter is an algebraic one: the maximal eigenvalue of the Laplacian. The second parameter is a topological one: the connectivity of the simplicial complex of independent sets. These two parameter are useful in studying combinatorial properties of the graph, such as the existence of independent transversals. In a previous work with Aharoni and Meshulam, we showed that these two parameters are related. In this talk we show that for claw-free graphs we can obtain bounds for these parameters that are better than the ones known for general graphs. similar results are obtained for $K_{1,k}$-free graphs for any $k$. Joint work with Noga Alon and Ron Aharoni.

**Tomaž Pisanski** (Univerza v Ljubljani and Univerza na Primorskem). *An interplay between topological graph theory and synthetic biology.*

Abstract. In 2013 a group of Slovenian scientists and mathematicians under the leadership of Roman Jerala produced a polypeptide string that can self-assemble in the shape of a stable tetrahedron in such a way that each tetrahedral edge is composed of two intertwined polypeptide segments. We will give a mathematical interpretation of this remarkable bioengineering task. The model that best describes a self-assembly polyhedron comes from topological graph theory. This model has been adequately described in a recent paper by G. Fijavž, T. Pisanski, J. Rus: Strong Traces Model of Self-Assembly Polypeptide Structures, MATCH Commun. Math. Comput. Chem. 71(2014) 199–212. However, there are several interesting question remaining. The model can be interpreted, on the one hand, as a gluing process turning a fundamental polygon into a closed surface and on the other hand, as an Eulerian trail in a doubled skeleton graph of the corresponding polyhedron. The design of such polypeptides requires a solution of several interesting combinatorial problems and problems of combinatorial optimization. Several questions concerning fundamental polygons are addressed. (Supported in part by the ARRS and ESF). Joint work with Nino Bašič, Roman Jerala and Martin Škoviera.

**Lowell Abrams** (The George Washington University). *Freely $\mathbb{Z}_n$-symmetric graphs, $\mathbb{Z}_n$-actions on the sphere, and excluded orbit minors.*
Abstract. Given a graph $G$ equipped with fixed-point-free $\Gamma$-action ($\Gamma$ a finite group), define an orbit minor $H$ of $G$ as a minor of $G$ for which the deletion and contraction sets are closed under the $\Gamma$-action.The orbit minor $H$ inherits a $\Gamma$-symmetry from $G$; furthermore, when the contraction set is acyclic the action inherited by $H$ remains fixed-point free. When $G$ embeds in the sphere and the $\Gamma$-action on $G$ extends to a $\Gamma$-action on the entire sphere, say that $G$ is $\Gamma$-spherical. We provide a complete catalog, for every odd value of $n \geq 3$, of the orbit-minor-minimal graphs $G$ with a free $\mathbb{Z}_n$-action that are not $\mathbb{Z}_n$-spherical. The case corresponding to $n = 1$ is Wagner's result from 1937 that the minor-minimal graphs that are not embeddable in the sphere are $K_5$ and $K_{3,3}$. Joint work with Dan Slilaty.

**Yuval Roichman** (Bar-Ilan University). *Flip graphs and group actions.*
Abstract. Many combinatorial objects, such as matchings, triangulations and trees carry a natural flip action. It will be shown that often the flip operation may be interpreted as a group (or monoid) action. This interpretation will be applied to estimate the radius and diameter of the associated flip graph and to enumerate geodesics between antipodes. Joint work with Adin, Athanasiadis, Elizalde and Firer, and theses of Avni, Ben-Ari and Khachatrian

**Eran Nevo** (Ben-Gurion University). *On graphs embedded in surfaces.*
Abstract. We consider two problems.
(1) Degree-diameter problem: What is the maximum number of vertices in graphs embedded is a given surface, with bounded diameter and degree? The embeddability condition allows to improve on the trivial Moore bound, and we show that the answer is similar to the situation for trees, in a precise sense. Even for planar graphs of odd diameter, better (almost tight) upper bounds are obtained.
(2) Bipartite minors: We introduce a notion of minors for bipartite graphs, and prove the following analog of Kuratowski-Wagner theorem: A bipartite graph is planar iff it does not contain $K_{3,3}$ as a bipartite-minor. Are there similar characterizations for bipartite graphs embedded in other surfaces? Joint work with (1) Guillermo Pineda-Villavicencio and David Wood, (2) Maria Chudnovsky, Gil Kalai, Isabella Novik and Paul Seymour.

## 21. The Mathematics of Menahem M. Schiffer

**Peter L. Duren** (University of Michigan). *Max Schiffer and his Variational Methods.*
Abstract. The purpose of this expository talk is to give an overview of Max Schiffer's contributions to geometric function theory. Following a brief

account of his scientific career, the discussion will focus on Schiffer's development of variational methods and their increasingly broad applications.

**Dov Aharonov** (Technion). *The Lebedev-Milin Inequalities.*
Abstract. The Lebedev-Milin inequalities are completely elementary in nature but very far from being obvious. In our talk we survey the very remarkable role these inequalities play in geometric function theory.

**Stephen J. Gardiner** (University College Dublin). *Potatoes and Potentials.*
Abstract. A "potato" is a compact connected subset of Euclidean space which is solid (that is, it coincides with the closure of its interior, and has connected complement) and has a uniform mass distribution. The background to this talk is the paper "Potato kugel" by Aharonov, Schiffer and Zalcman (Israel J. Math., 1981) and a sequel "Some inverse problems of potential theory" by Zalcman (Contemp. Math., 63, Amer. Math. Soc., 1987). If a potato $P$ induces an exterior gravitational field identical to that of a point mass, it is known that $P$ must be a ball. On the other hand, there exist distinct potatoes, with unit density (say), whose gravitational potentials agree near infinity. Zalcman has conjectured that this non-uniqueness phenomenon cannot arise if one of the potatoes is convex. This talk will survey subsequent developments related to this conjecture, which remains open.

**Dmitry Khavinson** (University of South Florida). *Isoperimetric Sandwiches" and Function Theory.*
Abstract. I shall discuss several classical isoperimetric inequalities (e.g., volume vs. surface area of the boundary, harmonic radius vs. volume radius, torsional rigidity of a domain vs. area) as corollaries of so-called "isoperimetric sandwiches" that involve seemingly unrelated function-theoretic and operator-theoretic quantities: analytic and harmonic contents (best approximations), norms of self-commutators of some simple subnormal operators, etc. I will illustrate this theme with a few recent examples and then focus on several attractive open problems.

**Nir Lev** (Bar-Ilan University). *Equidistribution Estimates for Fekete Points on Complex Manifolds.*
Abstract. We discuss extremal point configurations, called Fekete points, on a compact complex manifold. Their equidistribution is a result due to Berman, Boucksom and Witt Nyström. We propose another approach to the result based on the relation of Fekete points to another array of points, the sampling and interpolation points. This approach allows us to estimate the equidistribution of the Fekete points quantitatively. Joint work with Joaquim Ortega-Cerdá.

**Shahar Nevo** (Bar-Ilan University). *Differential Polynomials and Shared Values.*
Abstract. Let $f$ and $g$ be non-constant meromorphic functions in $\mathbb{C}$, $a$ and $b$ non-zero complex numbers and let $n$ and $k$ be natural numbers satisfying $n \geq 5k+17$. Using Nevanlinna Theory, we show that if the differential polynomials $f^n + af^{(k)}$ and $g^n + ag^{(k)}$ share the value $b$ counting multiplicities, then $f$ and $g$ are either equal or at least closely related. Joint work with Jürgen Grahl.

## 22. PDEs: Modeling Theory and Numberics

**Steve Schochet** (Tel-Aviv University). *Singular Limits Involving Variable-Coefficient Large Operators.*
Abstract. The classical theory of singular limits for evolution equations assumes that the large terms have constant coefficients. In recent years a number of results have been proven concerning hyperbolic systems having large variable-coefficient terms. After reviewing earlier results some more recent progress will be described.

**Boris Ettinger** (Princeton University). *Local well-posedness for the equation of minimal hypersurface in Minkowski space.*
Abstract. A timelike minimal hypersurface in Minkowski space satisfies a quasilinear wave equation. I will explain how the minimal hypersurface equation exhibits a null structure and how to utilize the null structure in order to lower the regularity requirements on the initial data for the Cauchy problem.

**Yehuda Pinchover** (Technion). *Optimal Hardy-type inequality for non-negative second-order elliptic operator: an answer to a problem of Shmuel Agmon.*
Abstract. We give a general answer to the following fundamental problem posed by Shmuel Agmon 30 years ago:

> *Given a (symmetric) linear elliptic operator $P$ of second-order in $\mathbb{R}^n$, find a continuous, nonnegative weight function $W$ which is "as large as possible" such that for some neighborhood of infinity $\Omega_R$ the following inequality holds*

$$(P\phi, \phi) \geq \int_{\Omega_R} W(x)|\phi|^2 \, dx \qquad \forall \phi \in C_0^\infty(\Omega_R).$$

We construct, for a *general* subcritical second-order elliptic operator $P$ in a domain $\Omega \subset \mathbb{R}^n$ (or a noncompact manifold), a Hardy-weight $W$ which is *optimal* in the following natural sense sense. The operator $P - \lambda W$ is subcritical in $\Omega$ for all $\lambda < 1$, null-critical in $\Omega$ for $\lambda = 1$, and supercritical near any neighborhood of infinity in $\Omega$ for any $\lambda > 1$. Moreover, in the symmetric case, if $W > 0$, then the spectrum and the essential spectrum of $W^{-1}P$ are

equal to $[1, \infty)$. Our method is based on the theory of positive solutions and applies to both symmetric and nonsymmetric operators on a general domain $\Omega$ or on a noncompact manifold. Moreover, the results can be generalized to certain $p$-Laplacian type operators. The constructed weight $W$ is given by an explicit simple formula involving two positive solutions of the equation $Pu = 0$.

**Eduard Feireisl** (Academy of Sciences of the Czech Republic). *Weak solutions to quantum hydrodynamics equations.*
Abstract. We consider solvability and related questions to certain systems arising in quantum hydrodynamics and semiconductor modeling. In particular, the well posedness in the class of weak solutions and the weak-strong uniqueness property are discussed. The method of convex integration is adapted to the degenerate situation in the presence of "vacuum".

**Dan Mangoubi** (Hebrew University). *Harmonic functions on the lattice: Absolute monotonicity, three circles theorems, construction.*
Abstract. We reveal an absolute monotonicity property associated with harmonic functions which strengthens a three circles theorem on harmonic functions due to Agmon. Then, we show how this property can be naturally discretized, and deduce three circles theorems for discrete harmonic functions.Furthermore, we describe a new algorithm to transform harmonic functions in $\mathbb{R}^d$ to discrete harmonic functions in $\mathbb{Z}^d$. Joint work with Gabor Lippner.

**Ram Band** (Technion). *Neumann Nodal Domains.*
Abstract. Laplacian eigenfunction on a two-dimensional manifold dictates some natural partitions of the manifold; the most apparent one being the well studied nodal domain partition. An alternative partition is revealed by considering a set of distinguished gradient flow lines of the eigenfunction - those which are connected to saddle points. These give rise to Neumann nodal domains. We define Neumann nodes and Neumann nodal domains and present their fundamental topological properties. These in turn allow to discuss some aspects of counting the number of Neumann nodal domains, giving estimates on their geometry and connecting them to the 'usual' nodal domains. Joint work with David Fajman, Peter Kuchment, Mark Dennis and Alexander Taylor.

**Gadi Fibich** (Tel-Aviv University). *Continuations of the NLS beyond the singularity.*
Abstract. The NLS is the canonical model for propagation of intense laser beams in transparent media. It has been known since 1965 that that the NLS model breaks down when the input power is sufficiently high, so that the solution undergoes collapse. The standard approach for continuing the

solution beyond the singularity has been to consider a more comprehensive model, in which the collapse is arrested. Motivated by the theory of vanishing-viscosity solutions in shock-wave theory, we adopt a different approach and ask whether singular NLS solutions can be continued beyond the singularity, within the framework of the NLS model. In this talk I will present some potential continuations of singular NLS solutions, and discuss whether any of them is "physical". A universal property of all the continuations is that after the singularity, the continued solution loses its initial phase information. This phase-loss phenomenon implies that the relative phase between two post-collapse filaments becomes random. In other words, one cannot deterministically control the interactions between post-collapse filaments. Recent experimental confirmation of the phase-loss phenomenon will be presented.

**Alina Chertock** (North Carolina State University). *Convergence of a Particle Method and Global Weak Solutions for a Family of Evolutionary Pdes.*
Abstract. We provide global existence and uniqueness results for a family of fluid transport equations by establishing convergence results for the particle method applied to these equations. The considered family of PDEs is a collection of strongly nonlinear equations which yield traveling wave solutions and can be used to model a variety of fluid dynamics. The equations are characterized by a bifurcation parameter $b$, which provides a balance for the nonlinear solution behavior, and a kernel $G(x)$, which determines the shape of the traveling wave and the length scale. For some special cases of $b$ and $G(x)$, the equations are completely integrable and admit solutions that are nonlinear superpositions of traveling waves that have a discontinuity in the first derivative at their peaks and therefore are called peakons. Joint work with e apply a particle method to the considered evolutionary equations and provide a new self-contained method for proving its convergence. The latter is accomplished by using the concept of space-time bounded variation and the associated compactness properties. From this result, we prove the existence of a unique global weak solution to the family of fluid transport equations for $b > 1$ and a particular choice of $G(x)$ and obtain stronger regularity properties of the solution than previously established.

**Yoel Shkolnisky** (Tel-Aviv University). *A class of Laplacian mutiwavelets bases for high-dimensional data.*
Abstract. We introduce a framework for representing functions defined on high-dimensional data. In this framework, we propose to use the eigenvectors of the graph Laplacian to construct a multiresolution analysis on the data. This results in a one parameter family of orthogonal bases, which includes both the Haar basis as well as the eigenvectors of the graph Laplacian. We describe a discrete fast transform for expansion in any of the bases in

this family, and derive an asymptotic rate of coefficients decay. We demonstrate our construction using several numerical examples. Joint work with Nir Sharon.

**Gil Ariel** (Bar-Ilan University). *Parareal methods for highly oscillatory ordinary differential equations.*
Abstract. We introduce a multiscale parareal method that numerically integrates highly oscillatory ordinary differential equations. The algorithm computes a low-cost approximation of all slow variables in the system. Then, fast phase-like variables are computed in parallel using the parareal iterative methodology. The numerical scheme does not require that the system is split into slow and fast coordinates. Moreover, the dynamics may involve hidden slow variables, for example, due to resonances. Joint work with Seong Jun Kim and Richard Tsai.

**Peter Constantin** (Princeton University). *Long time behavior for forced 2D SQG.*
Abstract. We prove the existence of a compact finite dimensional global attractor for the forced critical surface quasi-geostrophic equation (SQG) in $\mathbb{T}^2$. To the best of our knowledge this is the first proof of finite dimensionality in a critical quasilinear equation. The global regularity of the forced critical SQG equation is revisited, with a new and final proof. We show that the system loses infinite dimensional information, by obtaining strong long time bounds that are independent of initial data, a fact that is quite nontrivial for critical equations. Joint work with V. Vicol and A. Tarfulea.

**Guy Katriel** (ORT Braude College). *Equilibrium distributions of some kinetic exchange models.*
Abstract. Kinetic exchange models, describing transfers of 'wealth' among randomly interacting agents, have been investigated in recent years in the field of 'econophysics'. Of particular interest are the equilibrium distributions of wealth to which these processes tend after a long time. In many cases an explicit form for these equilibrium distributions is unknown. In this work we identify some processes for which the equilibrium distribution can be explicitly found. Results ensuring the convergence of the time-dependent process to these equilibria will also be discussed.

**Vered Rom-Kedar** (Weizmann Institute). *New Lagrangian diagnostics for characterizing fluid flow mixing.*
Abstract. Quantifying fluid mixing is a long-standing scientific challenge, with important practical mplications (e.g., larval and pollutants dispersion). Lagrangian Coherent Structures (LCS) have been used to visualize and analyze mixing properties of flows and maps. We propose a new kind of Lagrangian diagnostic family by which the spatial structure of extreme values

of an observable are monitored. A specific form of it is suggested for characterizing mixing: the maximal extent of a trajectory (MET). This new diagnostic enables the detection of coherent structures and their dynamics in two- (and potentially three-) dimensional unsteady flows in both bounded and open domains. Moreover, besides being an intuitive diagnostics, it computation seems much easier compared with all other Lagrangian diagnostics known to us. It provides new insights regarding the mixing properties on both short and long time scales and on both spatial plots and distribution diagrams. The usefulness and applicability of this diagnostic to two dimensional flows is demonstrated using toy models and a data set of surface currents from the Mediterranean Sea. Joint work with R. Mundel, E Fredj and H. Gildor.

**Eitan Tadmor** (University of Maryland). *On the two-dimensional pressure-less Euler equations.*
Abstract. We prove the existence of weak solutions for the two-dimensional pressure-less Euler equations. To this end we develop an L1 framework of dual solutions for such equations. Their existence is realized as vanishing viscosity limits. Their limit follows from new BV estimates, derived by tracing the spectral dynamics of the velocity gradient matrix.

**László Székelyhidi** (University of Leipzig). *The h-principle and turbulence.*
Abstract. It is well known since the pioneering work of Scheffer and Shnirelman that weak solutions of the incompressible Euler equations exhibit a wild behaviour, which is very different from that of classical solutions. Nevertheless, weak solutions in three space dimensions have been studied in connection with a long-standing conjecture of Lars Onsager from 1949 concerning anomalous dissipation and, more generally, because of their possible relevance to the K41 theory of turbulence.

**Raz Kupferman** (Hebrew University). *Weizenbock manifolds as a homogenization limit of Riemannian manifolds with defects.*
Abstract. Materials with continuous distributions of defects are commonly modeled as smooth manifolds endowed with a metric and a non-symmetric metrically-compatible connection - i.e., the defects manifest as torsion. In the material science literature, the torsion is associated with the perturbation of crystal symmetries. In this lecture we will show that torsion arises naturally also in the absence of any crystalline symmetry. We prove a geometric homogenization theorem: a flat manifold with a non-symmetric connection can be obtained as a limit of Riemannian manifolds with an increasingly large number of cone defects.

**Marshall Slemrod** (University of Wisconsin). *An Analysis of Thermal Creep via Korteweg Theory.*

Abstract. In recent work the Slemrod has suggested that an exact sum of the Chapman-Enskog expansion for the Boltzmann equation yields constitutive relations similar to Korteweg's theory of capillarity. But if this conjecture is reasonable it needs to be checked in comparison with both solutions of the Boltzmann equation and experiments for rarefied gases. One such classical example is thermal creep where J.C. Maxwell (and Osborne Reynolds) in 1879 recognized that classical gas dynamics was insufficient in to make theoretical predictions for a rarefied gas driven by boundary thermal gradients. In this talk I will present exact and numerical solutions showing that Korteweg theory unlike classical Navier-Stokes-Fourier theory delivers results consistent with both the Boltzmann equation of kinetic theory and experiment.

**Edriss S. Titi** (Weizmann Inst and U of California - Irvine). *On the regularization mechanism for periodic dispersive equations: The Majida-Biello coupled KdV system paradigm.*

Abstract. In this talk we show the global well-posedness in $L^2$ of a coupled Korteweg-de Vries (KdV) system, subject to periodic boundary conditions. This system was introduced by Majda and Biello, describing nonlinear resonant interaction of Rossby waves. Our approach is based on a successive time-averaging method developed by Babin, Ilyin and Titi (2011) for the classical KdV equation, with periodic boundary conditions. Joint work with K. Simon and Y. Guo.

## 23. Contributed Papers (misc. topics)

**Dong Hyun Cho** (Kyonggi University). *Analogues of Wiener integrals with drift and initial distribution on a function space.*

Abstract. Let $C[0,t]$ denote the space of continuous real-valued functions on the interval $[0,t]$ and let $w_\varphi$ be an analogue of Wiener measure on the Borel class $\mathcal{B}(C[0,t])$ of $C[0,t]$, where $\varphi$ is a probability measure on the Borel class of $\mathbb{R}$. Let $h \in L_2[0,t]$ be of bounded variation with $h \neq 0$ a.e. on $[0,t]$ and $a$ be a continuous function on $[0,t]$. Define $X_0(x) = x(0)$ and a stochastic process $Z : C[0,t] \times [0,t] \to \mathbb{R}$ by $Z(x,s) = \int_0^s h(u)dx(u) + x(0) + a(s)$ for $x \in C[0,t]$ and $s \in [0,t]$, where the integral denotes the Paley-Wiener-Zygmund stochastic integral. In this talk we investigate the distribution of $Z$ and prove that $Z$ is a generalized Brownian motion process if $X_0$ is degenerated. We also establish a generalized Wiener integration theorem which extends the Wiener integration theorem in the classical Wiener space $C_0[0,t]$, where $C_0[0,t]$ is the space of continuous real-valued functions $x$ with $x(0) = 0$. Furthermore we derive a generalized Paley-Wiener theorem which generalizes the Paley-Wiener theorem in $C_0[0,t]$. As applications of these theorems we evaluate generalized Wiener integrals of various functions of

the forms

$$Z(x, s_1)Z(x, s_2)\cdots Z(x, s_m),\ \exp\bigg\{\int_0^t Z(x, s)dm_L(s)\bigg\},$$

and

$$\int_0^t (Z(x, s))^m dm_L(s), \int_{L_2[0,t]} \exp\{i(v, x)\}d\sigma(v)$$

for $x \in C[0, t]$, where $0 < s_1 < s_2 < \cdots s_m \leq t$, $m$ is a positive integer, $m_L$ is the Lebesgue measure on the Borel class of $\mathbb{R}$ and $\sigma$ is a complex Borel measure on $L_2[0, t]$. We note that these functions including the time integrals are important in the Feynman integration theories and quantum mechanics.

**Alejandro Velez-Santiago** (UC Riverside). *Quasi-linear variable exponent elliptic and parabolic problems with Wentzell boundary conditions.*
Abstract. Let $p \in C^{0,1}(\overline{\Omega})$ be such that $1 < p_* \leq p^* < \infty$, let $\Omega \subseteq \mathbb{R}^N$ be a bounded $W^{1,p(\cdot)}$-extension domain, and let $\mu$ be an upper $d$-Ahlfors measure supported on $\Gamma := \partial\Omega$ with $d \in (N - p_*, N)$. We investigate the solvability of a class of quasi-linear elliptic and parabolic boundary value problems involving the $p(\cdot)$-Laplace operator $\Delta_{p(\cdot)}$, and Wentzell boundary conditions

$$\Delta_{p(\cdot)}u\, d\mu - \Delta_{p(\cdot),\Gamma}u + |\nabla u|^{p(\cdot)-2}\frac{\partial u}{\partial \nu}d\mathcal{H}^{N-1} + \beta|u|^{p(\cdot)-2}u\, d\mu \;=\; 0 \qquad \text{on } \Gamma,$$

where $\beta \in L^\infty(\Gamma)$ is such that $\inf\limits_{x\in\Gamma} \beta(x) \geq \beta_0$ for some constant $\beta_0 > 0$, and $\Delta_{p(\cdot),\Gamma}$ denotes the $p(\cdot)$-Laplace-Beltrami operator over $\Gamma := \partial\Omega$. We first show that the elliptic problem admits a unique globally bounded solution. Secondly, we establish that the realization of the $p(\cdot)$-Laplace operator with the above boundary conditions generates a (nonlinear) order-preserving submarkovian $C_0$-semigroup on $L^2(\Omega) \times L^2(\Gamma)$, and hence, its associated first order Cauchy problems is well-posed on $L^{q(\cdot)}(\Omega) \times L^{q(\cdot)}(\Gamma)$ for all measurable function $q$ with $1 \leq q_* \leq q^* < \infty$. To conclude, a (nonlinear) ultracontractivity property for such semigroup is achieved, which implies as a consequence that mild solutions of the above boundary value problem are globally bounded.

**David Fajman** (Vienna). *The Einstein-flow in 2+1 dimensions.*
Abstract. The global structure of solutions to the Einstein equations can be analyzed by studying the evolution of geometric quantities under the Einstein flow. An approach to the general problem of understanding the global geometry of space times is the non-linear stability problem, which considers initial data close to explicitly known solutions and analyzes its future development.The stability question is essential in Mathematical Relativity and is only understood for few explicit solutions.¡br¿In the talk we present results on the stability of solutions to the Einstein equations in 2+1 dimensions in

the presence of Vlasov matter without symmetry assumptions. We discuss a technique to construct geometric energies for distribution functions of this type of matter and how these are used to prove stability. In addition, we construct future complete and stable solutions to the Einstein-flow on the 2-sphere - which is shown to be an exclusive feature of Vlasov matter in 2+1 dimensions.

**Avram Sidi** (Technion). *Recent Asymptotic Expansions for the Trapezoidal Rule, Gauss–Legendre Quadrature, and Legendre Polynomial Expansions.*
Abstract. In this talk, we will discuss some recent asymptotic results related to problems in numerical quadrature and approximation theory.

- We will present a generalization of the Euler–Maclaurin expansion for the trapezoidal rule approximation of finite-range integrals $\int_a^b f(x)dx$, when $f(x)$ is allowed to have arbitrary algebraic-logarithmic endpoint singularities.
- We will present a full asymptotic expansion (as the number of abscissas tends to infinity) for Gauss–Legendre quadrature for integrals $\int_a^b f(x)dx$, where $f(x)$ is allowed to have arbitrary algebraic-logarithmic endpoint singularities.
  If the trapezoidal rule and the Gauss–Legendre quadrature are applied following a suitable variable transformation, their accuracy can be improved dramatically despite the fact that $f(x)$ may (and almost always will) remain singular following the variable transformation. The variable transformations involved normally have endpoint singularities that can be tuned to optimize the performance of the quadrature formulas. We will illustrate this point numerically with the Gauss–Legendre quadrature.
- We will present full asymptotic expansions, as $n \to \infty$, of Legendre series coefficients $a_n = (n+1/2)\int_{-1}^{1} f(x)P_n(x)dx$, when $f(x)$ has arbitrary algebraic-logarithmic (interior and/or endpoint) singularities in $[-1, 1]$. In addition, we will give full asymptotic expansions, as $n \to \infty$, for $P_n(x)$, $|x| \leq \pi - \epsilon$, and for the integral $\int_c^d f(x)P_n(x)dx$, $-1 \leq c \leq d \leq 1$.
  These expansions are used to make statements about the asymptotic behavior, as $n \to \infty$, of the partial sums $\sum_{k=0}^{n} a_k P_k(x)$. This knowledge leads us to conclude that the Shanks transformation (or the equivalent epsilon algorithm of Wynn) and the Levin–Sidi $d$-transformation can be used very effectively to accelerate the convergence of the Legendre series $\sum_{k=0}^{\infty} a_k P_k(x)$ in question.

**Anahit Chubaryan and Armine Chubaryan** (Yerevan State U, Armenia). *Comparison of proof complexity for strongly equal tautologies in some*

*proof systems.*
Abstract. The research of the lengths of proofs in the systems of propositional calculus is important because of its relations with some main problems of the computational complexity theory: NP=co-NP?, NP=PSPACE? etc. One of the directions in propositional proof complexity investigations is to search hard provable tautologies for given proof system or to search such system, in which all tautologies have simple proof. We investigate some property of tautologies, which can characterize their proof complexity.
The traditional assumption that all propositional tautologies are equal to each other is not fine-grained enough to support a sharp distinction among tautologies. In fact, there are very simple tautologies, in which almost all variable are absolutely unless, while in the other tautologies nearly all variables are important. It is natural that the last tautologies are harder. We suggested earlier some idea to revise the notion of logical equivalence between classical tautologies in such way that it takes into account an appropriate measure of their complexity. We have introduced the notion of determinative conjunct, on the bas of which the notion of strong equality of classical tautologies was defined and have give the comparative analysis of strongly equal tautologies in some classical proof systems. Now on the base of determinative conjuncts and determinative disjunctive normal form, introduced by analogy for intuitionistic, minimal, monotone, fuzzy, modal (and some other) tautologies, we describe the partial hierarchy and strong equality for them in each logic. We compare also different measures (size, steps, space, width) of proof complexity for strongly equal tautologies in some week proof systems of above mentioned logic. We prove that 1) the strongly equal tautologies have the same proof complexities in some proof systems; and 2) there are such proof systems, in which some measures of proof complexities for strongly equal tautologies are the same, the other measures differ from each other only by the sizes of tautologies.
The study of the relations between the proof complexities of strongly equal tautologies in other more interesting systems of different logics is in progress. In particular we have some examples of most hard tautologies, which have exponential proof size in week systems, but their Frege proofs are polynomially bounded. If we can prove that all tautologies with the same hardness have the same Frege proofs, then we can solve the first of above mentioned problem. The systems, introduced on the base of determinative disjunctive normal form for different logics, can be the foundation for some universal propositional system construction.

**Iddo Ben-Ari** (University of Connecticut). *Diffusion with redistribution.*
Abstract. We consider a di ffusion process on a bounded domain with random redistribution, and study the nature of the resulting process. By redistribution we mean “jumping to new location according to some probability distribution on the domain, which may depend on the position of the path immediately prior to the redistribution. We consider two different trigger

mechanisms for the redistribution. The first, redistribution from the boundary, occurs when the process hits the boundary of the domain, and the second, instantaneous redistribution, occurs at jump times of a Poisson process, time-changed by the diffusion. Diffusion with redistribution appear in several applications, and we believe are theoretically interesting because of the non-trivial interaction between the fast (unbounded variation) local (continuous paths) and diffusive diffusion with the slow, nonlocal yet non-diffusive (keeps process in domain) redistribution. I will present recent results, work in progress and open problems for both mechanisms. For redistribution from the boundary, I will discuss ergodicity, spectral gap, but I will mostly focus on the problem of efficient coupling in one dimension. For the instantaneous redistribution model, I will discuss the asymptotic behavior of the model when the rate of the Poisson process tends to infinity, and the phase transition it exhibits.

**Moshe Klein** (Gan Adam LTD). *Dialogue in mathematics.*
Abstract. During the years we developed an innovative approach to mathematic education. The key idea is that mathematics may be studied from kindergarten age, by using a different approach. The reason for this is that kindergarten children have not begun to learn math at school in ordered fashion. Therefore, their thinking is different, creative, open and intuitive.
The math program is based on a dialogue approach in which the teacher learns together with the children. The children are enchanted by Math, learning the story of great mathematicians' life and discoveries. The children learn the concept of permutations, partitions and intrigued subjects related to the concept of number.
The Israeli Ministry of Education has taken an interest in the program and initiated a pilot program in several kindergartens.