The matched product of semi-braces

Paola Stefanelli

paola.stefanelli@unisalento.it

Università del Salento

Noncommutative and non-associative structures, braces and applications
Il-Gżira, 14th March 2018

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a (left) semi-brace if

$$a\circ(b+c) = a\circ b + a\circ\left(a^{-}+c\right),$$

holds for all $a,b,c\in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a \circ (b+c) = a \circ b + a \circ \left(a^- + c\right),$$

holds for all $a, b, c \in B$, where a^- is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B, +)$ is a left cancellative semigroup and $(B, \circ)$ is a group. We say that $(B, +, \circ)$ is a **(left) semi-brace** if

$$a \circ (b + c) = a \circ b + a \circ (a^{-} + c),$$

holds for all $a, b, c \in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B, +)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B, +)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a \circ (b+c) = a \circ b + a \circ (a^{-} + c),$$

holds for all $a, b, c \in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a\circ(b+c) = a\circ b + a\circ\left(a^{-}+c\right),$$

holds for all $a,b,c\in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a \circ (b + c) = a \circ b + a \circ \left(a^{-} + c\right),$$

holds for all $a, b, c \in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a \circ (b + c) = a \circ b + a \circ \left(a^{-} + c\right),$$

holds for all $a, b, c \in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a\circ(b+c)=a\circ b+a\circ\left(a^{-}+c\right),$$

holds for all $a,b,c\in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B,+)$ is a left cancellative semigroup and $(B,\circ)$ is a group. We say that $(B,+,\circ)$ is a **(left) semi-brace** if

$$a \circ (b + c) = a \circ b + a \circ \left(a^{-} + c\right),$$

holds for all $a, b, c \in B$, where a^{-} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B,+)$ a group, then B is a skew brace, [Guarnieri, Vendramin, 2017]. If in addition $(B,+)$ is abelian, then B is a brace [Rump, 2007].

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set

$$a + b := b \circ f(a),$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set
$$a + b := b \circ f(a),$$
for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set

$$a + b := b \circ f(a),$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set

$$a + b := b \circ f(a),$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set

$$a + b := b \circ f\,(a)\,,$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.

2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$. Set

$$a + b := b \circ f(a),$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r(a, b) = \left(a \circ (a^- + b), (a^- + b)^- \circ b \right)$$

*for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the **solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a(b) = a \circ (a^- + b)$,
- $\rho_b(a) = (a^- + b)^- \circ b$.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r(a, b) = \left(a \circ (a^- + b), (a^- + b)^- \circ b \right)$$

for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the ***solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a(b) = a \circ (a^- + b)$,
- $\rho_b(a) = (a^- + b)^- \circ b$.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r\left(a,b\right) = \left(\ a \circ \left(a^{-} + b\right),\ \left(a^{-} + b\right)^{-} \circ b\ \right)$$

*for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the **solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a\left(b\right) = a \circ \left(a^{-} + b\right)$,
- $\rho_b\left(a\right) = \left(a^{-} + b\right)^{-} \circ b$.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r(a, b) = \left(a \circ (a^- + b),\ (a^- + b)^- \circ b \right)$$

for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the ***solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a(b) = a \circ (a^- + b)$,
- $\rho_b(a) = (a^- + b)^- \circ b$.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r\left(a, b\right) = \left(\, a \circ \left(a^{-} + b\right), \left(a^{-} + b\right)^{-} \circ b \,\right)$$

*for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the **solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a\left(b\right) = a \circ \left(a^{-} + b\right)$,
- $\rho_b\left(a\right) = \left(a^{-} + b\right)^{-} \circ b$.

How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \to B \times B$ given by

$$r\left(a, b\right) = \left(\, a \circ \left(a^{-} + b\right), \left(a^{-} + b\right)^{-} \circ b \,\right)$$

*for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the **solution associated to the semi-brace B.***

In particular, if $a, b \in B$, then

- $\lambda_a\left(b\right) = a \circ \left(a^{-} + b\right)$,
- $\rho_b\left(a\right) = \left(a^{-} + b\right)^{-} \circ b$.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \operatorname{Aut}(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \operatorname{Aut}(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the matched system of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \operatorname{Aut}(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \operatorname{Aut}(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \operatorname{Aut}(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \operatorname{Aut}(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \operatorname{Aut}(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \operatorname{Aut}(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

such that

$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha^{-1}_{\beta_a(u)}(a)}$$

$$\lambda_u \beta_a = \beta_{\alpha_u(a)} \lambda_{\beta^{-1}_{\alpha_u(a)}(u)}$$

hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the **matched system** of B_1 and B_2 via α and β.

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$

$$(a, u) \circ (b, v) := \left(\alpha_u\left(\alpha_u^{-1}(a) \circ b\right), \beta_a\left(\beta_a^{-1}(u) \circ v\right)\right)$$

is a left semi-brace. We call this structure the matched product of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$

$$(a, u) \circ (b, v) := \left(\alpha_u \left(\alpha_u^{-1}(a) \circ b\right), \beta_a \left(\beta_a^{-1}(u) \circ v\right)\right)$$

is a left semi-brace. We call this structure the ***matched product*** *of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$
$$(a, u) \circ (b, v) := \left(\alpha_u \left(\alpha_u^{-1}(a) \circ b\right), \beta_a \left(\beta_a^{-1}(u) \circ v\right)\right)$$

is a left semi-brace. We call this structure the ***matched product*** *of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$

$$(a, u) \circ (b, v) := \left(\alpha_u\left(\alpha_u^{-1}(a) \circ b\right), \beta_a\left(\beta_a^{-1}(u) \circ v\right)\right)$$

*is a left semi-brace. We call this structure the **matched product** of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$
$$(a, u) \circ (b, v) := \left(\alpha_u \left(\alpha_u^{-1}(a) \circ b\right), \beta_a \left(\beta_a^{-1}(u) \circ v\right)\right)$$

*is a left semi-brace. We call this structure the **matched product** of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$
$$(a, u) \circ (b, v) := \left(\alpha_u \left(\alpha_u^{-1}(a) \circ b\right), \beta_a \left(\beta_a^{-1}(u) \circ v\right)\right)$$

*is a left semi-brace. We call this structure the **matched product** of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$
$$(a, u) \circ (b, v) := \left(\alpha_u\left(\alpha_u^{-1}(a) \circ b\right), \beta_a\left(\beta_a^{-1}(u) \circ v\right)\right)$$

is a left semi-brace. We call this structure the ***matched product*** *of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$
$$(a, u) \circ (b, v) := \left(\alpha_u \left(\alpha_u^{-1}(a) \circ b\right), \beta_a \left(\beta_a^{-1}(u) \circ v\right)\right)$$

is a left semi-brace. We call this structure the ***matched product*** *of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].

- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].

- Similarly, we obtain the matched product of two braces, as observed in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].

- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].

- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].

- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].

- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Some consequences

As a consequence of the result seen, we have:

- If B_1 and B_2 are semi-braces, $\alpha : B_2 \to \mathrm{Aut}\,(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semi-brace B_1, $\beta : B_1 \to \mathrm{Aut}\,(B_2)$ defined by $\beta_u := \mathrm{id}_{B_2}$, then $(B_1, B_2, \alpha, \beta)$ is a matched system of B_1 and B_2. In particular, the matched product of B_1 and B_2 via α and β is the **semidirect product** of B_1 and B_2 via α [Catino, Colazzo, P.S., 2017].
- Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces with B_1 and B_2 skew braces. Then the matched product of B_1 and B_2 (via α and β) is a skew brace, as obtained in [Smoktunowicz, Vendramin, 2018].
- Similarly, we obtain the matched product of two braces, as obtained in [Bachiller, 2018] and [Bachiller, Cedó, Jespers, Okniński, 2018].

Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If $(B_1, B_2, \alpha, \beta)$ is a matched system of semi-braces B_1 and B_2 via α and β, then

- *$(B_1, r_{B_1}, B_2, r_{B_2}, \alpha, \beta)$ is a matched product system of solutions.*
- *The solution $r_{B_1 \bowtie B_2}$ associated to $B_1 \bowtie B_2$ is exactly $r_{B_1} \bowtie r_{B_2}$.*

Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If $(B_1, B_2, \alpha, \beta)$ is a matched system of semi-braces B_1 and B_2 via α and β, then

- $(B_1, r_{B_1}, B_2, r_{B_2}, \alpha, \beta)$ *is a matched product system of solutions.*
- *The solution $r_{B_1 \bowtie B_2}$ associated to $B_1 \bowtie B_2$ is exactly $r_{B_1} \bowtie r_{B_2}$.*

Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If $(B_1, B_2, \alpha, \beta)$ is a matched system of semi-braces B_1 and B_2 via α and β, then

- *$(B_1, r_{B_1}, B_2, r_{B_2}, \alpha, \beta)$ is a matched product system of solutions.*
- *The solution $r_{B_1 \bowtie B_2}$ associated to $B_1 \bowtie B_2$ is exactly $r_{B_1} \bowtie r_{B_2}$.*

Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If $(B_1, B_2, \alpha, \beta)$ is a matched system of semi-braces B_1 and B_2 via α and β, then

- $(B_1, r_{B_1}, B_2, r_{B_2}, \alpha, \beta)$ *is a matched product system of solutions.*
- *The solution $r_{B_1 \bowtie B_2}$ associated to $B_1 \bowtie B_2$ is exactly $r_{B_1} \bowtie r_{B_2}$.*

Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If $(B_1, B_2, \alpha, \beta)$ *is a matched system of semi-braces* B_1 *and* B_2 *via* α *and* β*, then*

- $(B_1, r_{B_1}, B_2, r_{B_2}, \alpha, \beta)$ *is a matched product system of solutions.*
- *The solution* $r_{B_1 \bowtie B_2}$ *associated to* $B_1 \bowtie B_2$ *is exactly* $r_{B_1} \bowtie r_{B_2}$.

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e).$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e).$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B,+)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B,+)$ and E is the set of idempotents of $(B,+)$.
- $(G,\circ)$ and $(E,\circ)$ are groups and $(B,\circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e).$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e)$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e)$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B,+)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B,+)$ and E is the set of idempotents of $(B,+)$.
- $(G,\circ)$ and $(E,\circ)$ are groups and $(B,\circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e)$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e).$$

Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
- $(G, \circ)$ and $(E, \circ)$ are groups and $(B, \circ)$ is the matched product of these two groups.
- G is a skew brace and E is a trivial semi-brace.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

If B is a semi-brace, then B is isomorphic to the matched product of the skew brace G and the trivial semi-brace E.

In particular, α and β functions are defined by,

$$\alpha_e(g) := \left(\rho_{e^-}\left(g^-\right)\right)^-, \qquad \beta_g(e) := \lambda_g(e).$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1, e_1)}(g_2, e_2) = \left(\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2), \lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2, e_2)}(g_1, e_1) = \left(\rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta_{g_1}^{-1}(e_1)}(e_2)}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1) = \rho_{\alpha_{e_2}^{-1}(g_2)} \ldots$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha_{e_1}^{-1}(g_1)}(g_2)}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1, e_1)}(g_2, e_2) = \left(\lambda_{g_1} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2), \lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2, e_2)}(g_1, e_1) = \left(\rho_{\alpha_{e_2}^{-1}(g_2)} \alpha^{-1}_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta_{g_1}^{-1}(e_1)}(e_2)} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)} \alpha^{-1}_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1) = \rho_{\alpha_{e_2}^{-1}(g_2)} \alpha^{-1}_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha_{e_1}^{-1}(g_1)}(g_2)} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)} \beta^{-1}_{\lambda_{g_1} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1,e_1)}(g_2,e_2) = \left(\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2), \lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2,e_2)}(g_1,e_1) = \left(\rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta^{-1}_{g_1}(e_1)}(e_2)}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1) = \rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha^{-1}_{e_1}(g_1)}(g_2)}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1, e_1)}(g_2, e_2) = \left(\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2), \lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2, e_2)}(g_1, e_1) = \left(\rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta^{-1}_{g_1}(e_1)}(e_2)}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1) = \rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha^{-1}_{e_1}(g_1)}(g_2)}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1, e_1)}(g_2, e_2) = \left(\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2), \lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_2)}(e_2)\right)$$
$$\rho_{(g_2, e_2)}(g_1, e_1) = \left(\rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha_{\lambda_{\beta_{g_1}^{-1}(e_1)}(e_2)}^{-1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1) = \rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta_{\lambda_{\alpha_{e_1}^{-1}(g_1)}(g_2)}^{-1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1,e_1)}(g_2,e_2) = \left(\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2),\, \lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2,e_2)}(g_1,e_1) = \left(\rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta_{g_1}^{-1}(e_1)}(e_2)}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1) = \rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha_{e_1}^{-1}(g_1)}(g_2)}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1,e_1)}(g_2,e_2) = \left(\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2),\, \lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2,e_2)}(g_1,e_1) = \left(\rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1),\; 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta^{-1}_{g_1}(e_1)}(e_2)}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1) = \rho_{\alpha^{-1}_{e_2}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha^{-1}_{e_1}(g_1)}(g_2)}\beta_{\alpha^{-1}_{e_1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta^{-1}_{g_1}(e_1)}(g_2)}(e_1) = 0$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B.

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β, then the solution associated to B is the map $r : B \times B \to B \times B$ given by

$$\lambda_{(g_1,e_1)}(g_2,e_2) = \left(\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2),\, \lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_2)}(e_2)\right)$$

$$\rho_{(g_2,e_2)}(g_1,e_1) = \left(\rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1),\ 0\right)$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

$$\rho_{\alpha^{-1}_{\lambda_{\beta_{g_1}^{-1}(e_1)}}(e_2)\,\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1) = \rho_{\alpha_{e_2}^{-1}(g_2)}\alpha^{-1}_{\lambda_{e_1}\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}(g_1)$$

$$\rho_{\beta^{-1}_{\lambda_{\alpha_{e_1}^{-1}(g_1)}}(g_2)\,\beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}\beta^{-1}_{\lambda_{g_1}\alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}(e_1) = 0$$

Thanks for your attention!