

The matched product of semi-braces

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Noncommutative and non-associative structures, braces and applications

Il-Gżira, 14th March 2018

Semi-braces

In order to find new set-theoretical solutions of the Yang-Baxter equation, we introduce the algebraic structure of semi-brace.

Definition (F. Catino, I. Colazzo, and P.S., J. Algebra, 2017)

Let B be a set with two operations $+$ and $\circ$ such that $(B, +)$ is a left cancellative semigroup and $(B, \circ)$ is a group. We say that $(B, +, \circ)$ is a (left) semi-brace if

$$a \circ (b + c) = a \circ b + a \circ (a^{-1} + c),$$

holds for all $a, b, c \in B$, where a^{-1} is the inverse of a with respect to $\circ$.

In particular, if B is a semi-brace with $(B, +)$ a group then B is a brace [Guarnieri, Vendramin, 2017]. If in addition $(B, +)$ is abelian then B is a radical [Rump, 2007].

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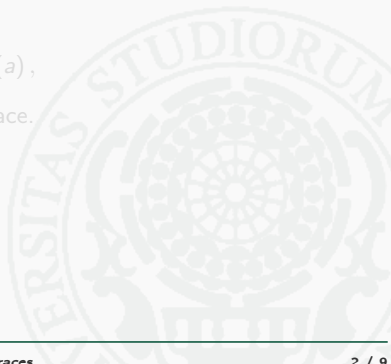
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Examples of semi-braces

1. If $(E, \circ)$ is a group, set $a + b = b$, for all $a, b \in E$, then $(E, +, \circ)$ is a semi-brace. We call E the **trivial semi-brace**.
2. If $(B, \circ)$ is a group and f is an endomorphism of $(B, \circ)$ such that $f^2 = f$.
Set

$$a + b := b \circ f(a),$$

for all $a, b \in B$, then $(B, +, \circ)$ is a semi-brace.

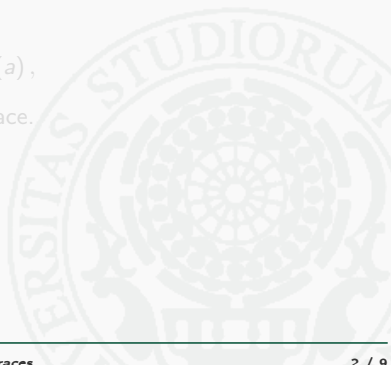


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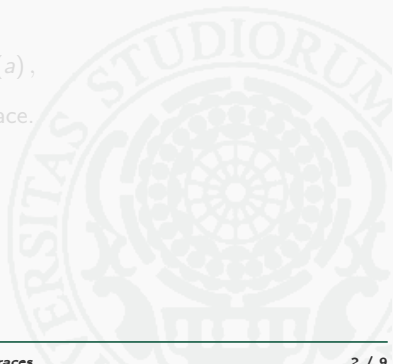


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How to obtain a solution through a semi-brace

Theorem (F. Catino, I. Colazzo, P.S., J. Algebra, 2017)

Let B be a semi-brace. Then, the map $r : B \times B \rightarrow B \times B$ given by

$$r(a, b) = \left(a \circ (a^- + b), (a^- + b)^- \circ b \right)$$

for all $a, b \in B$, is a left non-degenerate solution of the Yang-Baxter equation. We call r the **solution associated to the semi-brace B** .

In particular, if $a, b \in B$, then

- ▶ $\lambda_a(b) = a \circ (a^- + b),$
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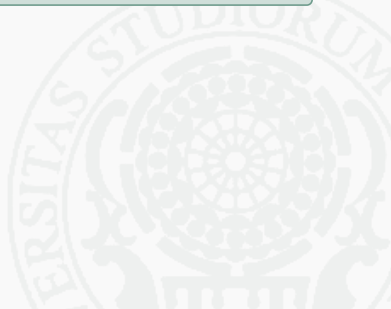
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The matched system of semi-braces

We introduce the matched system of semi-braces.

Definition (F. Catino, I. Colazzo, P.S., in preparation)

If B_1 and B_2 are semi-braces with

- ▶ $\alpha : B_2 \rightarrow \text{Aut}(B_1)$ a group homomorphism from the group $(B_2, \circ)$ into the automorphism group of the semigroup $(B_1, +)$,
- ▶ $\beta : B_1 \rightarrow \text{Aut}(B_2)$ a group homomorphism from the group $(B_1, \circ)$ into the automorphism group of the semigroup $(B_2, +)$,

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$$\lambda_a \alpha_u = \alpha_{\beta_a(u)} \lambda_{\alpha_{\beta_a(u)}^{-1}(a)}$$

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hold for all $a \in B_1$ and $u \in B_2$, then $(B_1, B_2, \alpha, \beta)$ is said to be the *matched system* of B_1 and B_2 via α and β .

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The construction

We introduce a new construction of semi-braces, the matched product of semi-braces.

Theorem (F. Catino, I. Colazzo, P.S., in preparation)

Let $(B_1, B_2, \alpha, \beta)$ be a matched system of semi-braces. Then the cartesian product $B_1 \times B_2$ with respect to the sum and the multiplication defined by

$$(a, u) + (b, v) := (a + b, u + v)$$

$$(a, u) \circ (b, v) := (\alpha_v(\alpha_u^{-1}(a) \circ b), \beta_a(\beta_v^{-1}(u) \circ v))$$

*is a left semi-brace. We call this structure the **matched product** of left semi-braces B_1 and B_2 (via α and β) and we denote it by $B_1 \bowtie B_2$.*

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Matched products of semi-braces and solutions

The solution associated to any matched product of semi-braces is of a particular kind.

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Solutions of semi-braces - I

Matched product construction turns out to be useful to obtain a description of every semi-brace. If B is a semi-brace,

- ▶ the semigroup $(B, +)$ is a right group, i.e., $B = G + E$, where G is a subgroup of $(B, +)$ and E is the set of idempotents of $(B, +)$.
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Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B .

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β , then the solution associated to B is the map $r : B \times B \rightarrow B \times B$ given by

$$\begin{aligned} \lambda_{(g_2, e_2)}(g_1, e_1) &= \left(\lambda_{g_2} \alpha_{\beta_{e_2}^{-1}(e_1)}(g_1), \lambda_{e_2} \beta_{\alpha_{e_1}^{-1}(g_2)}(e_1) \right) \\ \rho_{(g_2, e_2)}(g_1, e_1) &= \left(\rho_{\alpha_{e_2}^{-1}(g_2)} \alpha_{\lambda_{e_2} \beta_{\alpha_{e_1}^{-1}(g_2)}(e_1)}^{-1}(g_1), 0 \right) \end{aligned}$$

for all $e_1, e_2 \in E$ and $g_1, g_2 \in G$.

In particular,

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$$\begin{aligned}\rho_{\alpha_{\lambda_{\beta_{g_1}^{-1}(e_1)}(e_2)} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)} \alpha_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}^{-1}(g_1) &= \rho_{\alpha_{e_2}^{-1}(g_2)} \alpha_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}^{-1}(g_1) \\ \rho_{\beta_{\lambda_{\alpha_{e_1}^{-1}(g_1)}(g_2)} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)} \beta_{\lambda_{g_1} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2)}^{-1}(e_1) &= 0\end{aligned}$$

Solutions of semi-braces - II

We can view in a different light the solution associated to any semi-brace B .

Corollary (F. Catino, I. Colazzo, P.S., in preparation)

If $B := G \bowtie E$ is a semi-brace obtained as the matched product of G and E by α and β , then the solution associated to B is the map $r : B \times B \rightarrow B \times B$ given by

$$\begin{aligned}\lambda_{(g_1, e_1)}(g_2, e_2) &= \left(\lambda_{g_1} \alpha_{\beta_{g_1}^{-1}(e_1)}(g_2), \lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_2)}(e_2) \right) \\ \rho_{(g_2, e_2)}(g_1, e_1) &= \left(\rho_{\alpha_{e_2}^{-1}(g_2)} \alpha_{\lambda_{e_1} \beta_{\alpha_{e_1}^{-1}(g_1)}(e_2)}^{-1}(g_1), 0 \right)\end{aligned}$$

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Thanks for your attention!

