PRIMITIVE ALGEBRAS WITH ARBITRARY GELFAND-KIRILLOV DIMENSION

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ABSTRACT. We construct, for every real $\beta \geq 2$, a primitive affine algebra with Gelfand-Kirillov dimension β. Unlike earlier constructions, there are no assumptions on the base field. In particular, this is the first construction over $\mathbb{R}$ or $\mathbb{C}$.

Given a recursive sequence $\{v_n\}$ of elements in a free monoid, we investigate the quotient of the free associative algebra by the ideal generated by all non-subwords in $\{v_n\}$.

We bound the dimension of the resulting algebra in terms of the growth of $\{v_n\}$. In particular, if $|v_n|$ is less than doubly-exponential then the dimension is 2. This also answers affirmatively a conjecture of Salwa [12].

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1. PRELIMINARIES

Let A be an affine k-algebra. The **Gelfand-Kirillov dimension** [6] of A is defined as

$$GKdim(A) = \limsup_{s\to\infty} \frac{\log \dim(V + V^2 + V^3 + \cdots + V^s)}{\log s}$$

where V is a finite-dimensional subspace that generates A as an algebra. (see [9] for details).

It is easily seen that $GKdim(A) = 0$ iff A is finite dimensional. Otherwise $GKdim(A) \geq 1$, and by Bergman's gap theorem [3], either $GKdim(A) = 1$ (in which case A is a PI-algebra by [14]), or $GKdim(A) \geq 2$.

If A is PI then $GKdim(A)$ equals the transcendence degree of A over k [2], and is thus an integer.

Affine algebras with GKdim arbitrary real $\beta \geq 2$ were constructed by Borho-Kraft [4] and by Warfield [15] (*cf.* also [9, 2.9]). These examples fail to be prime. For a semiprime example, see [7].

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In [8] Irving and Warfield constructed primitive algebras with arbitrary GKdim, under the restriction that the base field F has an infinite-dimensional algebraic extension.

In this note we provide straightforward examples of primitive affine algebras over an arbitrary field, having arbitrary GKdim≥ 2.

Our construction is a generalization of the Morse algebra. It is a monomial algebra, that is, the quotient of a free associative algebra by an ideal generated by monomials. The growth of such algebras has been studied, for example, in [1], [5] and [11].

We assume the ideal to be generated by all monomials that are not subwords in a given sequence $\{v_n\}$ of elements in a free monoid, and relate properties of the resulting algebra A to the sequence $\{v_n\}$.

In section 2 we prove that under conditions (1), (2) below, $GKdim(A)$ is bounded in terms of $\frac{\log|v_{n+1}|}{\log|v_n|}$. If $|v_n|$ is less then doubly-exponential (as is the case if v_n is defined by a constant recursion rule), then $GKdim(A) \leq 2$. Some theory of recurring sequences over finite fields is used in section 3 to choose $\{v_n\}$ that achieve the bound, thus producing prime algebras of arbitrary GKdim. These examples are shown to be primitive in section 4.

2. MONOMIAL ALGEBRAS

Let $\mathcal{S}$ be a free finitely-generated monoid. If L is an ideal of $\mathcal{S}$, then $A = k[\mathcal{S}/L] \cong k[\mathcal{S}]/k[L]$ is an affine **monomial algebra**.

Fix a group $\mathcal{T}$ of permutation automorphisms of $\mathcal{S}$. If $M < \mathcal{S}$ is a submonoid, let $M_{\mathcal{T}}$ denote the closure of M under the operation of $\mathcal{T}$.

Let $\{v_n\}$ be a sequence in $\mathcal{S}$. Evidently, the set L of words that are not subwords in any v_n, is an ideal. In this case {subwords of length s in v_n: all n} is a basis for the s'th homogeneous part of A.

All algebras discussed below are defined over a fixed (but arbitrary) field k, with ideals L as above, where we assume

$$\forall n : |v_n| \leq |v_{n+1}| \tag{1}$$

$$\forall n : v_{n+\kappa} \in < v_n, v_{n+1}, \dots, v_{n+\kappa-1} >_{\mathcal{T}} \tag{2}$$

for some fixed κ.

Write $x \leq y$ if x is a subword in y. As in Salwa [12], there is an obvious criterion for A to be prime.

Remark 2.1. *A is prime iff for any i, j there exist $w \in \mathcal{S}$ such that $v_i w v_j < v_k$ for some k.*

If $x \in \mathcal{S}$, $X \subseteq \mathcal{S}$, let $W_s(x) = \{w : w \leq x, |w| = s\}$ and $W_s(X) = \bigcup_{x \in X} W_s(x)$. Obviously, $|W_s(x)| < |x|$ if $s > 1$.

Write $w_s = |W_s(\{v_n\})|$. Then

$$GKdim(A) = \limsup_{s \to \infty} \frac{\log(w_1 + w_2 + \cdots + w_s)}{\log s}.$$

Note that $GKdim(A) = 0$ iff $|v_n|$ is bounded. We assume henceforth that this is *not* the case, so by Bergman's gap theorem $GKdim(A) = 1$ or $GKdim(A) \geq 2$.

The main tool we use to compute $GKdim(A)$ is the following simple lemma.

Lemma 2.2. *If $|x_2|, |x_3|, \ldots, |x_{m-1}| \geq s$, then*

$$W_s(x_1 x_2 \ldots x_m) = W_s(x_1 x_2) \cup W_s(x_2 x_3) \cup \cdots \cup W_s(x_{m-1} x_m).$$

Proof. A subword of length s of $x_1 x_2 \ldots x_m$ can never intersect more then two consecutive x_i's. □

Theorem 2.3. *Let $d = \limsup \frac{\log |v_{n+1}|}{\log |v_n|}$. Then $GKdim(A) \leq 1 + d^\kappa$.*

Proof. Fix s and some $\epsilon > 0$.

There is some μ such that $|v_\mu| < s \leq |v_{\mu+1}|$. Iterating assumption (2), we get $v_{\mu+i} \in < v_{\mu+1}, \ldots, v_{\mu+\kappa} >_{\mathcal{T}}$ for all $i \geq 1$. Thus, by Lemma 2.2,

$$\begin{aligned} w_s &= |W_s(\{v_{\mu+i} : i \geq 1\})| \\ &\leq |\bigcup_{\tau_1, \tau_2 \in \mathcal{T}, 0 < j_1, j_2 \leq \kappa} W_s(\tau_1(v_{\mu+j_1}) \tau_2(v_{\mu+j_2}))| \\ &\leq \sum_{\tau_1, \tau_2 \in \mathcal{T}, 0 < j_1, j_2 \leq \kappa} |W_s(\tau_1(v_{\mu+j_1}) \tau_2(v_{\mu+j2}))| \\ &< \sum_{\tau_1, \tau_2 \in \mathcal{T}, 0 < j_1, j_2 \leq \kappa} |\tau_1(v_{\mu+j_1}) \tau_2(v_{\mu+j2})| \\ &\leq 2|\mathcal{T}|^2 \kappa^2 |v_{\mu+\kappa}| \end{aligned}$$

Let $c = 2|\mathcal{T}|^2 \kappa^2$. We have that $w_1 + w_2 + \cdots + w_s < cs|v_{\mu+\kappa}|$, so $\frac{\log(w_1 + \cdots + w_s)}{\log s} < \frac{\log cs}{\log s} + \frac{\log |v_{\mu+\kappa}|}{\log |v_\mu|} \leq 1 + d^\kappa + \epsilon$ for large enough s.

□

If $|v_n|$ is less then doubly exponential, *i.e.*, $\log \log |v_{n+1}| - \log \log |v_n| \to 0$, then $d = 1$ and $GKdim(A) \leq 2$.

It can be shown that $GKdim(A) = 1$ iff for some constant C, almost all the words v_n are periodic with period $< C$. We omit the details of the proof.

In many natural examples v_n are defined recursively. In this case we have

Corollary 2.4. *Suppose that $\{v_n\}$ is defined by a constant recursion rule (i.e. the formula for v_n as a function of $v_{n-1}, \dots, v_{n-\kappa}$ does not involve n), such that assumption (1) is satisfied.*

Then $|v_{n+1}| < M|v_n|$ for some constant M, and by Theorem 2.3 we have that $GKdim(A) \leq 2$.

In particular Salwa's example [12] has Gelfand-Kirillov dimension 2.

We end this section with

Lemma 2.5. *Assume that for any i, $v_i < v_k$ for some k. Then $GKdim(A) \geq 1 + limsup\frac{\log w_s}{\log s}$.*

Proof. The assumption implies that w_s is nondecreasing. Now

$$\begin{aligned} GKdim(A) &= limsup\frac{\log(w_1 + \cdots + w_{2s})}{\log 2s} \\ &\geq limsup\frac{\log s w_s}{\log 2s} \\ &= 1 + limsup\frac{\log w_s}{\log s}. \end{aligned}$$

□

3. Prime affine algebras with arbitrary dimension

In this section we present sequences $\{v_n\}$ that define prime algebras with arbitrary $GKdim > 2$. These examples are shown to be primitive in Section 4.

Some preliminaries from the theory of linear recurring sequences are needed. The reader is referred to [10, Chap. 8] for more details and proofs.

Proposition 3.1. *Let $m \geq 1$ be a natural number. Let K be the field of order 2^m. Pick a generator u of the multiplicative group K^*. Let $g(x) = g_0 + g_1 x + \cdots + x^m$ be the minimal polynomial of u over $\mathbb{Z}_2$.*

Define a sequence $\{b_i\}$ over $\mathbb{Z}_2$ by $b_0 = \cdots = b_{m-2} = 0, b_{m-1} = 1$, and the recursion rule $b_{i+m} = g_0 b_i + g_1 b_{i+1} + \cdots + g_{m-1} b_{i+m-1}$ $(i \geq 0)$.

Then $\{b_i\}$ has period $2^m - 1$, and for every non-zero $w \in \mathbb{Z}_2^m$, there is a unique $0 \leq i < 2^m - 1$ such that $w = b_i b_{i+1} \dots b_{i+m-1}$.

Moreover, if w and w' are opposite non-zero words (i.e. $w + w' = (11\dots1)$) of length $m+1$, then exactly one of them appears in $\{b_i\}$ ($(11\dots1), (00\dots0)$ do not appear at all).

Definition 3.2. *L_m denotes a word of length $2^m + m - 1$ over $\mathbb{Z}_2$, constructed as the first $2^m + m - 2$ elements of a sequence defined as in Proposition 3.1, preceded by a single* 0.

For example, we can take $L_1 = 01$, $L_2 = 00110$, $L_3 = 0001011100$ and $L_4 = 0000100110101111000$.

Remark 3.3. *Every word of length m appears exactly once as a subword of L_m. Two opposite words of length $m+1$ do not appear both in L_m except for the couple $(00\dots01),(11\dots10)$. No opposite words of length $m+2$ appear in L_m.*

Let $\mathcal{S} =< x, y >$ be the free monoid on two generators, with the automorphism $v \mapsto \bar{v}$ defined by $\bar{x} = y, \bar{y} = x$. The *substitution* of a word v in L_m is defined as the replacement of all 0's in L_m by v and all 1's by $\overline{v}$. For example, $L_1(v) = v\overline{v}$.

Fix a sequence of integers r_n. We define $\{v_n\} \subseteq \mathcal{S}$ as follows:

$$v_1 = x, \quad v_{n+1} = L_{r_n}(v_n).$$

Define an algebra A using $\{v_n\}$ as in the beginning of section 2. Note that assumptions (1) and (2) are satisfied (with $\kappa = 1$).

Note that if $r_n > 1$ then v_{n+1} starts with $v_n v_n$ and ends with v_n. The case $r_n = 1$ gives the well known Morse algebra.

Theorem 3.4. *A is prime.*

Proof. By Remark 2.1 we must show for any v_i, v_j that $v_i w v_j < v_k$ for some k and a word w. Pick $n = max\{i, j\}$, then $v_i, v_j < v_n$, so pick w_i, w_j such that $v_i w_i$ is a tail of v_n and $w_j v_j$ a head of v_n.

If $r_n > 1$ then $v_i w_i w_j v_j < v_n v_n < v_{n+1}$ by definition of L_{r_n}. Otherwise suppose $r_n = 1$; if $r_{n+1} > 1$ then $v_i w_i \overline{v_n} w_j v_j < v_{n+1} v_{n+1} < v_{n+2}$, and if $r_{n+2} = 1$ then $v_i w_i \overline{v_n}\,\overline{v_n} w_j v_j < v_{n+1} \overline{v_{n+1}} = v_{n+2}$. □

From now on we assume that $r_n \geq 3$.

Lemma 3.5. *If $m \geq (r_n + 2)|v_n|$, then the subwords of length m in v_{n+1} and in $\overline{v_{n+1}}$ are all different.*

Proof. Let $k = r_n + 2$. It is enough to prove the assertion in the case $m = k|v_n|$. For $n = 1$ the result follows from 3.3. Let $n > 1$.

Let a, b be two equal subwords of v_{n+1} or $\overline{v_{n+1}}$. Write v_{n+1} as a word on the letters $v_n, \overline{v_n}$ which we call *full letters.* Then a, b are determined by the full letter (v_n or $\overline{v_n}$) in v_{n+1} or in $\overline{v_{n+1}}$ in which they start, and the relative position in this full letter. The strategy is to show first that a and b start at the same relative position, and then show that they actually start at the same full letter.

Write $a = a_0u_1 \dots u_{k-1}a_1$ where $|a_0|, |a_1| \leq |v_n|$, and each u_i equals one of the full letters $v_n, \overline{v_n}$; write $b = b_0w_1 \dots w_{k-1}b_1$ in the same manner. *W.l.o.g.* we assume $|a_0| \geq |b_0|$.

Write $a_0 = a_{00}a_{01}$ and $b_1 = b_{10}b_{11}$ where $a_{00} = b_0$ and $b_{11} = a_1$. Also factor $u_i = u_i'u_i''$ and $w_i = w_i''w_i'$ where $|u_i''| = |w_i''| = |a_0| - |b_0|$.

Assume $|a_0| - |b_0| \leq \frac{1}{2}|v_n|$ (the other case is treated similarly). Then $u_1' = w_1'$ is an equality of words of length $\geq \frac{1}{2}|v_n|$.

Since $\frac{1}{2}|v_n| \geq \frac{r_{n-1}+2}{2^{r_{n-1}}+r_{n-1}-1}|v_n| = (r_{n-1}+2)|v_{n-1}|$, the induction hypothesis force u_i', w_i' to begin in the same relative position. But then it follows that u_i'', w_i'' are empty words and each of u_i', w_i' is a full letter, v_n or $\overline{v_n}$. By Remark 3.3, the $r_n + 1$ equalities $u_i = u_i' = w_i' = w_i$ force one of two cases: a, b begin in the same position in v_{n+1} or in $\overline{v_{n+1}}$, in which case we are done, or $u_1 \dots u_{k-1} = w_1 \dots w_{k-1} = v_n \dots v_n\overline{v_n}$ and $a \leq v_{n+1}, b \leq \overline{v_{n+1}}$ (or *vice versa*). But $v_n \dots v_n\overline{v_n}$ is the header of v_{n+1}, so a_0, b_0 must be empty. Then we have that $a_1 = b_1$, the $(r_n + 2)$'th equality of full letters, a contradiction of Remark 3.3. □

We can now compute the Gelfand-Kirillov dimension of A.

Theorem 3.6. *Let* $d = \limsup \frac{r_1+\cdots+r_n}{r_1+\cdots+r_{n-1}}$. *Then* $GKdim(A) = d + 1$.

Proof. Note that $|v_n| = |L_{r_1}||L_{r_2}| \dots |L_{r_{n-1}}|$, so

$$r_1 + \cdots + r_{n-1} < \log_2 |v_n| < n + r_1 + \cdots + r_{n-1}.$$

By Theorem 2.3 we have

$$GKdim(A) \leq 1 + \limsup \frac{\log_2 |v_{n+1}|}{\log_2 |v_n|} = 2 + \limsup \frac{\log_2 |L_{r_n}|}{\log_2 |v_n|} \leq$$

$$\leq 2 + \limsup \frac{r_n + 1}{r_1 + \cdots + r_{n-1}} = 1 + d.$$

For the other direction, recall that by Lemma 3.5 all of the subwords of length $s = (r_n + 2)|v_n|$ in v_{n+1} are different. Thus

$$w_s \geq |v_{n+1}| - s = (2^{r_n} - 3)|v_n|$$

(where w_s is the number of subwords of length s in any v_n), and, if $d > 1$,

$$\frac{\log_2 w_s}{\log_2 s} > \frac{\log_2(2^{r_n} - 3) + \log_2 |v_n|}{\log_2(r_n + 2) + \log_2 |v_n|} > \frac{r_1 + \cdots + r_n - 1}{\log_2(r_n) + n + r_1 + \cdots + r_{n-1}}.$$

The *limsup* of the lower bound is d (whether d is finite or infinite). If $d = 1$, then the expression in the middle already apporaches 1. □

Finally, let $\beta \in \mathbb{R}$, $\beta \geq 1$.

Take $r_n = \max([\beta^n], 3)$, and define an algebra A as above. Checking the conditions of Theorem 3.6, we arrive at

Theorem 3.7. *A is an affine prime algebra with $GKdim(A) = \beta + 1$.*

In particular, the bound in 2.3 is tight (at least for $\kappa = 1$).

4. Our examples are primitive

In this section we show that A is primitive. We assume that $r_n \geq 3$, and $r_n > 3$ infinitely often (note that for dimension 2 we must take $r_n = 4$).

Definition 4.1. *For $u, v \in \mathcal{S}$, let $u \leq_l v$ ($u \leq_r v$) denote that u is a head (tail) of v. An element $a \in A$ is a* **left** *(***right***)* **tower** *if the set of monomials of a is linearly ordered by $\leq_l$ ($\leq_r$).*

Being a left tower is invariant under multiplication by a monomial from the left.

Lemma 4.2. *Let $L < A$ be a left ideal. Then L contains a left tower.*

Moreover, for every $a \in A$, wa is a left tower for some monomial w.

Proof. It is enough to show that if a_1, a_2 are monomials, and $wa_1 = 0$ iff $wa_2 = 0$ (all $w \in \mathcal{S}$), then a_1, a_2 are $\leq_l$-comparable. Assume $|a_1| \geq |a_2|$.

For some n we have $a_2 \leq v_n$ and $r_n > 3$. Write $v_n = \alpha a_2 \beta$, $v_{n+1} = uv_n$.

By assumption $u\alpha a_1 \neq 0$, so $u\alpha a_1 \leq v_m$ for some m. Writing v_m as a word in the letters $v_{n+1}, \overline{v_{n+1}}$, the intersection of u with some letter is of length $> \frac{1}{2}|u| > (r_n+2)|v_n|$. By Lemma 3.5, u (and thus $u\alpha a_1$) appears in v_m as a header of v_{n+1}, and we get $u\alpha a_2 \leq_l u\alpha a_1$, as desired. □

Corollary 4.3. *Let $I \trianglelefteq A$ be an ideal. Then I contains a monomial.*

Proof. By Lemma 4.2 and left-right symmetry, there is some $a \in I$ that is a left and right tower.

Let u, w be two different monomials in a, $|u| \leq |w|$. Multiplying by long enough monomials from both sides we may assume that $w = v_n$, and $\frac{1}{2}|v_n| < |u|$.

Now $u \leq_r v_n$ and $u \leq_l v_n$ implies by Lemma 3.5 that $u = v_n$, a contradiction. □

Let $J = < x, y > \trianglelefteq A$, a maximal ideal in A.

Recall that if $J_1, \dots J_n \trianglelefteq R$ are maximal ideals in a (unital) ring R, and $J_1 J_2 \dots J_n \subseteq I \neq R$, then $I \subseteq J_i$ for some i (for taking a maximal ideal $I_1 \supseteq I$ we have some $J_i \subseteq I_1$, implying $J_i = I_1 \supseteq I$).

Corollary 4.4. *J is the unique maximal ideal in A. Moreover, for any $I \trianglelefteq A$, the quotient A/I is a finite dimensional k-algebra and $Jac(A/I) = J/I$.*

Proof. By Corollary 4.3, $v_n \in I$ for some n. Now every monomial of length $> 2|v_{n+1}|$ contains v_{n+1} or $\overline{v_{n+1}}$ and thus v_n as a subword, so $J^m \subseteq I$ for $m = 2|v_{n+1}|$, and thus $I \subseteq J$.

A/I is finite dimensional spanned by $\{$ words of length $< m$ $\}$, and $Jac(A/I) = Jac(A/J^m)/(I/J^m) = (J/J^m)/(I/J^m) = J/I$. □

Corollary 4.5. *The only prime ideals of A are $0, J$.*
In particular, the (classical) Krull dimension of A is one.

Proof. 0 is prime by Remark 2.1. If $I \neq 0$ is prime then $J/I = Jac(A/I) = 0$ so $I = J$. □

Theorem 4.6. *A is primitive.*

Proof. Assume, on the contrary, that A is non-primitive. Then the only primitive ideal of A is J, so this is the Jacobson radical of A. In particular, $x + y \in J$ should be quasi-regular.

But if $a(1-x-y) = 1$, let w be a longest monomial in a. Necessarily $wx \neq 0$ or $wy \neq 0$, so wx (say) appears on the left hand side of the equality but not on the right hand side, a contradiction. □

From Corollary 4.4 it follows that A satisfies ACC on two-sided ideals (since a chain going up from $I \trianglelefteq A$ has length $< dim(A/I)$ as a k-algebra). For one-sided ideals the situation is not that pleasant.

Remark 4.7. *A is a not left-Noetherian.*

Proof. We construct a left ideal that is not finitely-generated.

Ordering the monomials in A by $<_r$ we get a tree T_1, in which every node is a root for a tree with at least two, and thus infinitely many, branches (indeed, if $w < v_n$ is a monomial write $v_n = \alpha w \beta$, then $w <_r v_n \alpha w$ and $w <_r \overline{v_n} \alpha w$). Now define a tree T_n and $w_n \in T_n$ ($n \geq 1$) as follows: Let $w \in T_n$ be a monomial with $wx, wy \in T_n$. Take $w_n = wx$, and T_{n+1} the tree with root wy.

Then $L = Aw_1 + Aw_2 + \dots$ is definitely not *f.g.*, since the w_i are $\leq_l$-incomparable. □

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